

WAVELENGTH DEPENDENCE OF DISPLACEMENT AND VELOCITY RESOLUTION IN LASER DOPPLER VIBROMETRY

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A theoretical approach is used to calculate wavelength dependence of displacement and velocity resolution in the technology of laser Doppler vibrometry (LDV). Mathematical description of a typical LDV system in the heterodyne arrangement is considered in this regard. Thermal noise and shot noise are assumed as the primary source of noise. Minimum target displacement and velocity which produce noise equivalent signal are considered as displacement and velocity resolution respectively. A theoretical linear relationship between laser wavelength and the mentioned resolutions is obtained when all other operational parameters of the LDV system are kept constant. The results of the present research are in good agreement with the earlier experimental researches conducted by others.

Keywords: laser Doppler vibrometry, displacement resolution, velocity resolution, laser wavelength, signal to noise ratio, noise equivalent signal.

СПЕКТРАЛЬНАЯ ЗАВИСИМОСТЬ ПРОСТРАНСТВЕННО-СКОРОСТНОГО РАЗРЕШЕНИЯ ЛАЗЕРНОЙ ДОПЛЕРОВСКОЙ ВИБРОМЕТРИИ

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Для расчета спектральной зависимости разрешения по скорости и по смещению в технологии лазерной доплеровской виброметрии (ЛДВ) используется теоретический подход. Рассматривается математическое описание типичной ЛДВ-системы в гетеродинном исполнении. В качестве основных источников шума принимаются тепловой и дробовой шум. Минимальные смещение и скорость мишени, которые генерируют сигнал, эквивалентный шуму, считаются разрешением по смещению и скорости. Между длиной волны лазера и указанными разрешениями имеет место линейная зависимость, если все другие рабочие параметры системы ЛДВ сохраняются постоянными. Полученные результаты хорошо согласуются с данными экспериментальных исследований других авторов.

Ключевые слова: лазерная доплеровская виброметрия, пространственное разрешение, разрешение по скорости, длина волны лазера, отношение сигнал/шум, эквивалентный шумовой сигнал.

Introduction. In recent years laser Doppler vibrometry (LDV) has gained fundamental importance in high precision, contactless, and remote motion measurements in scientific and industrial applications [1]. The technology of LDV was introduced at Columbia University where Yeh and Cummins [2] used infra-red helium-neon laser Doppler shift to measure fluid velocity. Since that time, LDV has exerted a remarkable impact on a wide range of industrial and medical applications [3]. The basic operation principle of LDV is based on Doppler frequency shift that occurs when the light is scattered by a vibrating object [4]. Because Doppler frequency shift is in the MHz range and laser frequency is in the terahertz range, detection of frequency shift is done interferometrically. Directional ambiguity can be removed by introducing a known frequency shift to either object or reference beams in the interferometry setup. By the action of this known fre-

quency shift through the Bragg cell, Doppler mechanism will induce a positive frequency shift to the incident light when the object is moving toward the emitting point, and a negative frequency shift when the object is moving away from the laser. Quadrature detection is another approach for direction discrimination in LDV technology [5], but the application of Bragg cell is the most common approach. Single probe and double probe beam LDV techniques are utilized for the measurement of translational vibrations of a point on the object and relative vibrations between two points on the target structure respectively. A scanning head can be attached to both single beam and double beam configurations to survey sequential points on the target surface. Vibration measurement in thin, hot, light, soft, or rotating structures with traditional contacting instruments is a challenging task while noncontact LDV technology has a practical advantage when dealing with this types of targets [1]. The main source of measurement error in the LDV system is located in the photodetector and subsequent electronic chain process in the amplifiers and filters [6]. However, some optical parameters such as laser wavelength and coherence length can affect measurement uncertainty of an LDV system. Measurement error in LDV systems was not of primary concern at the beginning of this technology, because it was small enough when compared to the other vibration analysis methods. However, with the extension of LDV utilization in scientific and industrial applications, measurement uncertainty becomes more important. The effects of wavelength variation on the resolution of LDV system is investigated in the present research. In order to concentrate on wavelength effects, we assumed that all other optical and electronic parameters of the system such as laser divergence and coherence length, splitters, plates, mirrors, photo detector response to incident light, etc. remain fixed when the wavelength is changing.

Modeling and calculations. *Theory of operation.* Doppler frequency shift arises from variations in the optical path from source to the detector when the target object is vibrating, as illustrated in Fig. 1. The incident laser field on the detector surface can be expressed by [7]:

$$\mathbf{E}_d = \mathbf{E}_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{s}_f - 2\mathbf{k} \cdot \mathbf{s}(t)), \quad (1)$$

where \mathbf{E}_0 is laser electric field amplitude, ω and \mathbf{k} are laser frequency and wave vector, \mathbf{s}_f is a distance which is passed by laser when the target is fixed, and $\mathbf{s}(t)$ is the vector of target displacement. Thus, $2\mathbf{k} \cdot \mathbf{s}(t) = (4\pi/\lambda)\mathbf{s}(t) = \varphi_D(t)$ can be considered as phase shift due to the target vibrations. LDV is based on the interferometrically combination of Doppler-shifted beam with a reference beam. Although both homodyne and heterodyne configuration of interferometry are used in LDV, the heterodyne method, which has an acousto-optic modulator for eliminating directional ambiguity, is the prevailing approach.

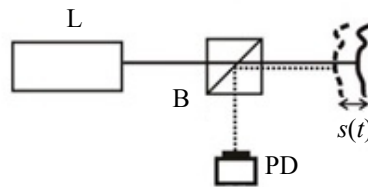


Fig. 1. Displacement in the target which produces Doppler shift [6]. L, B, PD, and $s(t)$ stand for the laser source, beam splitter, photodiode, and target displacement, respectively.

As illustrated in Fig. 2, the electric field at detector surface is

$$\mathbf{E}_d = \mathbf{E}_m \cos(\omega_m t - \mathbf{k} \cdot \mathbf{s}_{mf} - \varphi(t)) + \mathbf{E}_r \cos(\omega_r t - \mathbf{k} \cdot \mathbf{s}_r), \quad (2)$$

where \mathbf{E}_m and \mathbf{E}_r are measurement and reference beam electric field amplitudes, and ω_m , ω_r , \mathbf{s}_{mf} , and \mathbf{s}_r are measurement and reference beam angular frequency and traveled path, respectively. According to Eq. (2), the incident powers at the detector surface and generated current are

$$P_d = P_m + P_r + 2\sqrt{P_m P_r} \cos((\omega_m - \omega_r)t - \mathbf{k} \cdot \mathbf{s}_{mf} - \varphi(t) + \mathbf{k} \cdot \mathbf{s}_r), \quad (3)$$

$$i_d = K P_d = K [P_m + P_r + 2\kappa \sqrt{P_m P_r} \cos((\omega_m - \omega_r)t - \mathbf{k} \cdot \mathbf{s}_{mf} - \varphi(t) + \mathbf{k} \cdot \mathbf{s}_r)], \quad (4)$$

where P_m and P_r are measurement and reference beam powers, respectively; K stands for detector power to current conversion factor in A/W, and $0 < \kappa < 1$ indicates misalignment and distortion factor in both reference and measurement beams, which ideally can be considered as unity. Because information of target vibration is stored in the third term of Eq. (4), this term generates carrier signal in the technology of LDV. The decoding process can be carried on to extract phase shift and hence frequency shift due to the target vibrations.

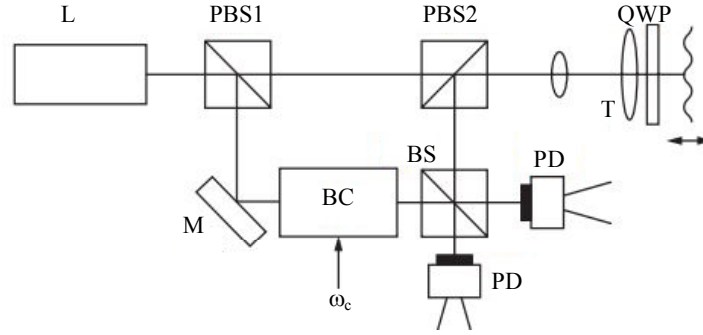


Fig. 2. Heterodyne configuration of LDV system [6]. L is the laser source, PBS1 and PBS2 are polarizer beam splitter, the symbol T is used to indicate the two-lens system for expanding and collimating laser beam, QWP indicates a quarter wave plate, M is used for mirror, BC is a Bragg cell acousto-optic modulator with modulation frequency of ω_c , BS is a beam splitter, and PD stands for photodetector. Two detectors are used to obtain full power of reference and measurement beams.

Noise analysis. Shot noise [8] and thermal noise [9] provide main error source in the LDV technology, and other types of noises such as flicker noise [10] and dark current noise [11] are not of primary concern when dealing with LDV systems. Shot noise is due to the random variations in the rate at which charge carriers are generated and recombined when the detector is impinged upon by incident beams. The equivalent shot noise current squared is represented by [9]

$$\bar{i}_{\text{shot}}^2 = 2qBK(P_m + P_r), \quad (5)$$

where q is electron charge and B is detector bandwidth. The thermal noise, which is sometimes called Johnson noise, is generated by thermal fluctuations of current in conducting materials. Moving electrons are constantly colliding with other electrons and atoms. Because electrons and atoms are randomly oscillating in their locations due to their thermal energy, the motion of electrons between them produces tiny random currents. Although integrating these currents over time results in zero, summing up these random fluctuations over short time intervals produces Johnson noise. The thermal noise equivalent current in square is

$$\bar{i}_{\text{th}}^2 = (4k_B T/R_L)B, \quad (6)$$

where K_B is Boltzmann constant, T is absolute temperature, and R_L is detector load resistance. A specific value for signal (Eq. (4)) to noise (Eq. (5) plus Eq. (6)) ratio is required for the intelligibility of signal. The signal to noise ratio is

$$SNR = \frac{EP_s}{EP_n} = \frac{\bar{i}_s^2}{(\bar{i}_{\text{shot}})^2 + (\bar{i}_{\text{th}})^2} = \frac{\eta \varepsilon^2 P_m P_r}{2qBK(P_m + P_r) + 4K_B T R_L^{-1} B}, \quad (7)$$

where EP_s and EP_n are the electric power of signal and noise, i_s is the signal current, and η and ε are quantum efficiency and efficiency factor of the detector. The Doppler frequency shift due to the vibrating target can be obtained by derivation of phase shift in Eq. (1) with respect to time:

$$\Delta f = \Delta\omega/2\pi = 2v(t)/\lambda = 2v_0/\lambda \cos(2\pi f_{\text{vib}}t + \varphi_{0v}). \quad (8)$$

We assumed that the motion of target is oscillatory with vibration frequency of f_{vib} and velocity amplitude of v_0 . In heterodyne configuration, the signal bandwidth equals twice the sum of Doppler shift and target vibration frequency. The bragg cell frequency, which is called central frequency, should be greater than half the signal bandwidth [10]:

$$B = 2(\Delta f + f_{\text{vib}}), \quad f_{\text{Bragg}} \equiv f_c \geq B/2 = \Delta f + f_{\text{vib}}, \quad (9)$$

where B is detector bandwidth in Eq. (7).

Signal decoding. Both analog and digital phase decoding procedures are commonly utilized in the technology of LDV [12]. The short delay time between input and output demodulated signal as well as applicability in high target vibration frequencies are advantages of analog decoding, while high precision and low noise level are superiorities of the digital method. Analog decoders utilize a phase locked loop (PLL) circuit [13]. The PLL circuit can detect a $\pm\pi/2$ phase shift in the input signal. Thus, according to the Eq. (1), we can write

$$\varphi_D(t)_{\min} = \pm\pi/2 = (4\pi/\lambda)s_{\min}(t) \rightarrow s_{\min}(t) = \pm\lambda/8. \quad (10)$$

Significant improvement in LDV resolution is achieved through the digital decoding method. In this approach the digital signal processing unit and an appropriate numerical procedure is used to decode detector input signal. The classical method of fringe counting is utilized in many commercially available LDV systems as a digital decoding approach, but it is no longer adequate for present precision requirements because the fringe counting method measures discrete increment of 2π in the input signal. Thus, it results in displacement resolution of $\pm\lambda/2$ for the LDV system. Continuous phase measurement methods [14] such as the arctangent phase demodulation method [15] lead to much higher displacement resolution. Numerical production of the i and q signal pair is the fundamental principle of this approach. Indeed, the i and q signal pair is created as

$$u_i = U_i \cos \varphi(t), \quad u_q = U_q \sin \varphi(t). \quad (11)$$

Each one of the pair signals contains information on displacement, and combination of both i and q signals results in direction discrimination. As illustrated in Fig. 3, i and q signals have 90° -phase shift relative to each other and may generate a complete phase circle for large target vibrations or incomplete phase arc for small vibrations. The I and Q signals are digitized by a pair of similar appropriate analog-to-digital converters. The calculated phase angle from digitized pair signal is

$$\varphi_D(t_n) = \arctg(u_q(t_n)/u_i(t_n)) + m\pi, \quad m = 0, 1, \dots \quad (12)$$

Similar to analog decoding in Eq. (8), the minimum detectable target displacement or LDV precision can be presented as

$$S_{\min}(t_n) = (\lambda/4\pi)\varphi_D(t_n)_{\min}. \quad (13)$$

Analog-to-digital converter sampling frequency determines minimum achievable phase angle according to the Eq. (10) and Fig. 3. This phase angle is known as detector digital decoding resolution [16]. When digital decoding resolution is $\pi/10$, the LDV resolution is equal to $S_{\min}(t_n) = \lambda/40$ based on Eq. (13).

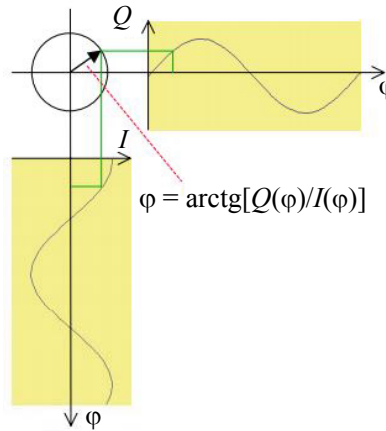


Fig. 3. i and q signals which have 90° shift with respect to each other [6].

Results and discussion. As mentioned above, to understand minimum detectable displacement and velocity of the target in LDV we have to analyze the significance of carrier signal and noise source. The detector current which is expressed by Eq. (4) is the sum of the DC current proportional to the sum of the measurement and reference powers and an AC component proportional to the geometrical average of measurement and reference powers. The AC term produces the power of carrier signal in LDV because it is the single term which has information of target vibrations. On the other hand, DC current provides shot noise in the system, as discussed above. In LDV systems, the laser power is usually selected such that shot noise provides a substantial contribution to the overall noise, and the system is called shot noise limited. In this sense Eq. (7) will be simplified to

$$SNR = \frac{EP_s}{EP_n} = \frac{\bar{i}_s^2}{\bar{i}_{\text{shot}}^2} = \frac{\eta \epsilon^2 P_m P_r}{h\nu B(P_m + P_r)}, \quad (14)$$

where h is Planck's constant and ν is laser frequency. Because the reference beam power is much greater than the measurement beam power, Eq. (14) can be approximated by

$$SNR = EP_s / EP_n = \bar{i}_s^2 / \bar{i}_{\text{shot}}^2 = \eta \epsilon^2 P_m / h\nu B. \quad (15)$$

In a given detector bandwidth, the carrier signal is presumed intelligible when $SNR \geq 1$. In other words, we should have noise equivalent signal at least. The sinusoidal carrier signal $u_c(\phi_c) = \hat{U}_c \sin \phi_c$ has a maximum rise of $du_c/d\phi_c = \pm \hat{U}_c$ in the vicinity of the zero crossings. Thus, if a single noise component with peak voltage \hat{U}_n is superimposed, the resulting maximum phase deviation is $\Delta\phi_n = \hat{U}_n / \hat{U}_c$. Because two uncorrelated noise components below and above the carrier frequency always contribute to noise modulation, a factor of $\sqrt{2}$ has to be taken into account. Consequently, a heterodyne carrier undergoes a peak phase deviation caused by the homogeneous spectral noise power distribution with respect to bandwidth, according to

$$\Delta\phi_n = 2\hat{U}_n / \hat{U}_c = \sqrt{2} \sqrt{EP_n / EP_c} = \sqrt{2} / \sqrt{SNR}, \quad (16)$$

where \hat{U}_n and \hat{U}_c are noise signal and carrier signal voltage amplitudes respectively, and EP_n and EP_c are standing for noise and carrier signals electric power respectively. FM demodulation is required when analyzing the velocity signal of Eq. (8). As is indicated in Eq. (8), the Doppler frequency shift is the result of the time derivative of phase shift induced by target vibration. So the frequency deviation is proportional to the vibration frequency of target:

$$\Delta f_n = f_n \Delta\phi_n = 2f_n \hat{U}_n / \hat{U}_c = \sqrt{2} f_n \sqrt{EP_n / EP_c} = f_n \sqrt{2} / \sqrt{SNR}. \quad (17)$$

In this sense Eq. (16) demonstrates that in a given fixed detector bandwidth displacement, the lower detection limit is independent of vibration frequency, whereas Eq. (17) indicates that the velocity lower detection limit is linearly proportional to the target vibration frequency. But from another point of view regarding Eq. (9), detector bandwidth is a function of target vibration frequency as well as Doppler frequency shift. Therefore by the increment in target vibration frequency, larger detector bandwidth is required. Consequently, based on Eqs. (5) and (6), thermal noise and shot noise are enhanced and signal-to-noise ratio is reduced. Then, an increment in displacement and velocity lower detection limit is expected when target vibration frequency is increased. For a typical LDV system when a 2 mW helium-neon laser is used in the set-up shown in Fig. 2 along with the 50% split ratio of beam splitters, quantum efficiency and efficiency factor of 0.8, lossless optical components, and target reflectivity of 10%, we get the following expressions for signal-to-noise ratio, phase deviation, and frequency deviation. According to Eqs. (14) and (17)

$$SNR = \frac{\eta \epsilon^2 P_m P_r}{h\nu B(P_m + P_r)} = \frac{0.8 \times 0.8^2 \times 0.1 \text{ (mW)} \times 1 \text{ (mW)}}{3.143 \times 10^{-19} \text{ (J)} B(0.1 \text{ (mW)} + 1 \text{ (mW)})} = \frac{1.57 \times 10^{14}}{B}, \quad (18)$$

$$\Delta\phi_n = \sqrt{2} / \sqrt{SNR} = 1.13 \times 10^{-7} \sqrt{B}, \quad (19)$$

$$\Delta f_n = f_n \Delta\phi_n = f_n \sqrt{2} / \sqrt{SNR} = 1.13 \times 10^{-7} f_n \sqrt{B}. \quad (20)$$

Target displacement and velocity resolution can be obtained according to Eqs. (19) and (20):

$$\Delta S_{\min} = (\lambda/4\pi) \Delta\phi_{\min} = 1.13 \times 10^{-7} (\lambda/4\pi) \sqrt{B}, \quad (21)$$

$$\Delta v_{\min} = (\lambda/2) \Delta f_{\min} = 1.13 \times 10^{-7} f_n (\lambda/2) \sqrt{B}. \quad (22)$$

The values of the signal-to-noise ratio $SNR = 1.57 \times 10^{14}$, phase deviation $\Delta\phi_n = 1.13 \times 10^{-7} \sqrt{B}$, frequency deviation $\Delta f_n = 1.13 \times 10^{-7} f_n \sqrt{B}$, displacement resolution $\Delta S_{\min} = 1.13 \times 10^{-7} (\lambda/4\pi) \sqrt{B}$, and target velocity resolution $\Delta v_{\min} = 1.13 \times 10^{-7} f_n (\lambda/2) \sqrt{B}$ for a typical heterodyne LDV described above.

Equation (21) demonstrates a theoretical displacement lower detection limit of $\approx 2 \times 10^{-15} \text{ m/Hz}^{1/2}$ for a typical heterodyne LDV described above with the helium-neon laser source. It is in good agreement with the results of experimental research where the displacement lower detection limit of $10^{-14} \text{ m/Hz}^{1/2}$ is reported. The lower displacement resolution is due to the lower detector quantum efficiency in that experimental work [17]. A displacement resolution of $\leq 10^{-12} \text{ m/Hz}^{1/2}$ is also reported by other researches [6, 9]. Additional noise sources such as speckle noise and spurious noise can decrease the resolution limit of real LDV systems, but these noise sources have minor importance when compared to the thermal noise and shot noise [9].

Figure 4 illustrate minimum displacement and velocity of the target object which produces noise equivalent signal with respect to the laser wavelength and detector bandwidth. In Fig. 4 the laser wavelength varies from 300 to 3000 nm because it is the wavelength range of commercially available lasers, and the interval of signal acquisition bandwidth is 10 to 40 MHz because it is the feasible interval for the heterodyne arrangement of the LDV system according to Eq. (9). The value of $f_n = 10 \text{ kHz}$ is considered for the target vibration frequency.

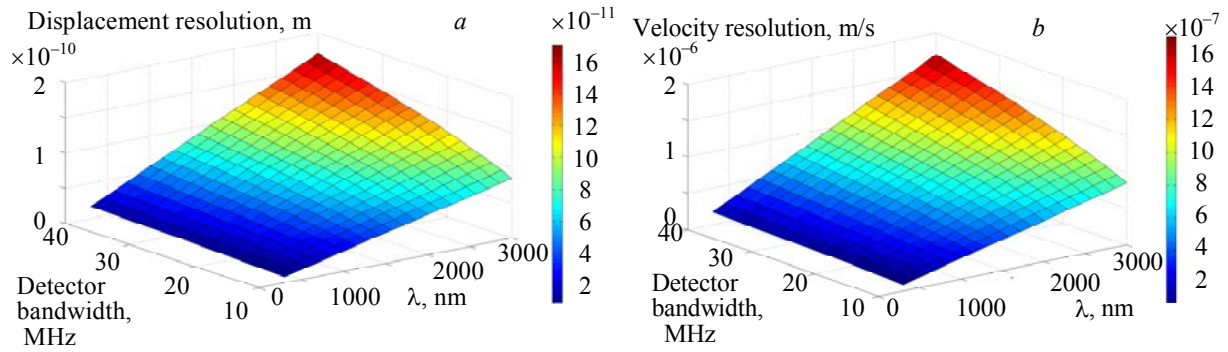


Fig. 4. Displacement resolution (a) and velocity resolution (b) in the heterodyne LDV system as a function of laser wavelength and detector bandwidth.

According to the Fig. 4a for a typical heterodyne LDV system described above, when a detector bandwidth of approximately 10 MHz and laser wavelength of about 300 nm is used, a displacement lower detection limit of a few tens of picometers is expected. Then by an increase in laser wavelength, the displacement lower detection limit will increase linearly. Growth in displacement resolution based on square root increment of detector bandwidth is also illustrated in Fig. 4a. Finally, for a detector bandwidth of approximately 40 MHz and laser wavelength of nearly 3000 nm, displacement resolution of a few tens of nanometers is depicted in Fig. 4a. As illustrated in Fig. 4b, a velocity resolution of a few hundred nanometers per second is accessible in the heterodyne LDV system when the laser wavelength is 300 nm and detector bandwidth is about 10 MHz. Linear and square root increment in the velocity lower detection limit are indicated based on the increase in the laser wavelength and detector bandwidth respectively, as shown in Fig. 4b. When the laser wavelength reaches the value of 3000 nm and the detector bandwidth is increased to the value of 40 MHz, the velocity resolution is about a few micrometers per second.

According to Eq. (22) the velocity resolution in an LDV system is linearly proportional to the target vibration frequency. In Fig. 5 the velocity resolution of the LDV system is presented with respect to the laser wavelength and target vibration frequency. The range of vibration frequency variations is chosen from 5 to 15 kHz because this interval is related to the human speech frequency which can be considered as a source of target vibration. The value of $B = 32$ MHz is assumed for the detector bandwidth in Fig. 5. Based on Fig. 5, a velocity resolution of submicrometer per second is achievable by a typical heterodyne LDV system when the detector bandwidth is 32 MHz, target vibration frequency is lower than 20 kHz, and laser wavelength is approximately 300 nm. The linear increment in the velocity lower detection limit from increase in both laser wavelength and target vibration frequency is evident in Fig. 5 when the detector bandwidth is considered fixed. As indicated in Fig. 5, when target vibration frequency is about 15 KHz, which is equivalent to the highest human speech frequency, and laser wavelength is approximately 3000 nm, the velocity resolution of a heterodyne LDV system with a detector bandwidth of 32 MHz is raised to several micrometers.

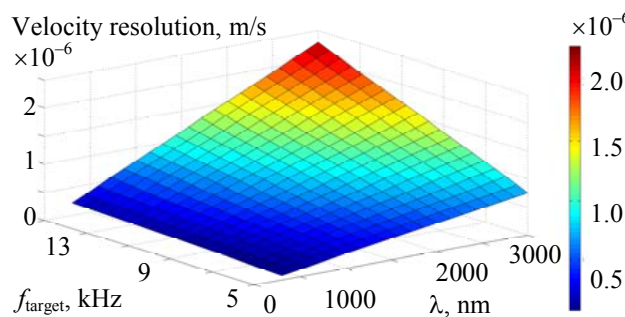


Fig. 5. Velocity resolution in the heterodyne LDV system as a function of laser wavelength and target vibration frequency.

Conclusion. A theoretical description of a system of laser Doppler vibrometry (LDV) in the heterodyne arrangement is presented. Shot noise and thermal noise are assumed as the main noise sources of the system. Analog and digital signal decoding approaches are introduced, and the digital signal decoding method is selected. Displacement resolution and velocity resolution for a noise equivalent signal are calculated. The outcomes of the present research demonstrate a linear relationship between minimum target displacement and velocity which produce noise equivalent signal and laser wavelength when all other operational parameters of the system are constant. The range of displacement resolution and velocity resolution are illustrated with respect to the laser wavelength and detector bandwidth. Target velocity resolution is also depicted with respect to the target vibration frequency. The results support experimental investigations of the resolution of LDV systems in the heterodyne arrangement. By the application of a standard milliwatt laser, noise limited picometer displacement resolution is accessible for the detector bandwidth of approximately 10 MHz. Also, nanometer displacement resolution is achievable for a detector bandwidth of up to 100 MHz. In addition, noise limited target velocity resolution of less than 1 micrometer per second is attainable through the application of a milliwatt laser for a target vibration frequency of several tens of kHz.

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