INTERPLAY BETWEEN SELF-FOCUSING AND SELF-COMPRESSION OF ELLIPTICAL *q*-GAUSSIAN LASER PULSE INTERACTING WITH AXIALLY INHOMOGENEOUS PLASMA**

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A theoretical investigation of spatiotemporal dynamics of an intense laser pulse with a q-Gaussian spatial irradiance profile interacting with collisionless plasma has been presented. In particular, the dynamics of pulse width and beam widths of the laser pulse have been investigated in detail. Using variational theory, nonlinear partial differential equation governing the evolution of the pulse envelope has been reduced to a set of coupled ordinary differential equations for the pulse width and beam widths of the laser pulse. The differential equations thus obtained have been solved numerically to envision the interplay between self-focusing and self-compression of the laser pulse.

Keywords: *partial differential equation, laser pulse, q-Gaussian.*

ВЗАИМНОЕ ВЛИЯНИЕ САМОФОКУСИРОВКИ И САМОСЖАТИЯ ЭЛЛИПТИЧЕСКОГО *q*-ГАУССОВА ЛАЗЕРНОГО ИМПУЛЬСА ПРИ ВЗАИМОДЕЙСТВИИ С НЕОДНОРОДНОЙ ВДОЛЬ ОСИ ПЛАЗМОЙ

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Теоретически изучена пространственно-временная динамика интенсивного лазерного импульса с q-гауссовым пространственным профилем излучения при взаимодействии с бесстолкновительной плазмой, в частности динамика ширины импульса и ширины луча лазерного импульса. С использованием вариационной теории нелинейное уравнение в частных производных, определяющее эволюцию огибающей импульса, сведено к набору связанных обыкновенных дифференциальных уравнений для ширины импульса и ширины луча лазерного импульса. Дифференциальные уравнения решены численно для представления взаимосвязи между самофокусировкой и самосжатием лазерного импульса.

Ключевые слова: *дифференциальное уравнение в частных производных, лазерный импульс, q-гауссиан.*

Introduction. Lasers have become an essential part of our life. They are ubiquitous in consumer technology, from CD players to supermarket checkout scanners [1]. The everyday presence of lasers does not mean that they are meant only for pedestrian tasks. Already they are providing the preferred solution to an impressive variety of real-world problems, and in the future, they will continue to enhance the quality of life and will contribute wealth to the world economy.

The vital organs of any laser system are optical amplification and feedback. In the late 1960s the rock musician, Jimi Hendrix, amazed audiences by placing his Stratocaster guitar in front of an amplifier to invoke a wail of sustained acoustic feedback. Earlier in the same decade, T. H. Maiman [2] of Hughes Research Laboratories did something similar with light. He demonstrated that an excited ruby rod placed in a cavity formed by two parallel mirrors can produce an intense beam of laser light. By bouncing light back

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and forth, the cavity provides optical feedback. One of the mirrors is partially reflecting and thus allows a portion of the light to escape to form the familiar laser beam. Many different light wavelengths can fit into the space between the mirrors, the only condition being that they must have an integral number of oscillations in the space between these mirrors. Each such electromagnetic wave is called a longitudinal mode of the cavity. These modes are the optical equivalent of the harmonic notes on each string of Hendrix's guitar. A well-controlled laser system operating in continuous wave mode emits a constant beam of light of a color corresponding to the frequency of one of these modes. However, a short laser pulse must contain a large range of frequencies, as mandated by Heisenberg's uncertainty principle [3, 4]. The frequency (or wavelength) content of a pulse is inversely proportional to the pulse width. To generate an ultrashort pulse, these many thousands of modes must be locked together in a phase [5]. When each mode is summed up with all the other modes, the sinusoidal electromagnetic waves add up to yield a short pulse in time, with a duration roughly equal to the inverse of the frequency range of the modes that are amplified in the cavity. As an analogy, imagine a row of bells of different tones, swinging in a bell tower. If the bells swing randomly, the result will be a steady cacophony. In contrast, if they swing in synchrony at regular intervals, they will produce a sequence of loud, equally spaced chords. Similarly, in mode-locked operation, all the light within the laser's optical cavity is confined to a discrete ultrashort pulse that bounces back and forth in the laser cavity, with a small amount of light leaking out through one mirror on each round trip. The laser thus emits a periodic sequence of ultrashort pulses through its partially transmissive mirror. In a physical sense, the term ultrashort characterizes a number of processes with evolution time scales that are smaller than the relaxation time for the medium. These pulses are brief enough that they can be used to take snapshots of phenomena occurring on molecular time scales, i.e., in the short time intervals during which molecules and electrons move and interact. Just as closely spaced flash photographs can be used to capture the nuances of a golfer's swing or a diver's entry into the pool; thus, scientists can use ultrashort pulses of light to capture in detail the consequences of the vibrational motion of the nuclei in a molecule or the flight of an electron during the brief moment before it scatters from the lattice of a semiconductor crystal.

When focused on a tiny spot, such laser pulses ablate many materials [6, 7]. This makes ultrashort pulses an efficient tool for micromachining, drilling, cutting, and welding. Precision can exceed the beam focus if the pulse intensity is set so carefully that the temperature of the material only at the brightest central part of the beam rises above the ablation threshold. The ultra-fast laser pulses ensure smooth and precise features by delivering energy to the target at the focus so rapidly that heat does not get enough time to diffuse into the surrounding unirradiated areas. This limits collateral damage and allows repeatable micron-sized cuts [8]. This feature of ultrashort laser pulses may prove practical in the decommissioning of weapons. The lasers can slice safely through high explosives and can vaporize the material at the cutting point without detonating the adjoining material. In the emerging surgical application, this means little if any burning and tearing of neighboring tissue. Unlike other laser surgery, which typically cauterizes an incision, cuts made with an ultrafast laser will bleed. The pulsed beam can also be focused beneath the skin, allowing for some types of incisionless surgery without damage to intervening tissue [9, 10]. In addition, cuts on the dimensions of microns may result in new capabilities for surgically repairing nerve damage.

The key to ultra-high-intensity laser pulses is a technique called chirped pulse amplification (CPA) [11, 12]. "Chirping" a signal or a wave means stretching it in time. In CPA, the first step is to produce a short pulse with an oscillator and stretch it, usually 1 to many times as long. This operation decreases the intensity of the pulse by the same amount. Standard laser amplification techniques can now be applied to this pulse. Finally, a sturdy device, such as a pair of diffraction gratings in a vacuum, recompresses the pulse to its original duration – increasing its power to several times beyond the amplifier's limit. However, it is easier said than done. The same is true with CPA-perfecting, which is not as straightforward as it sounds. The devices used to stretch/compress laser pulses generally do not do so in an exactly linear fashion. If the characteristics of the chirper are same to that of the compressor then the result gets spoiled. Further, the finite bandwidth of the active medium and the thermal damage threshold of the conventional gratings limit the peak intensity achievable by CPA. As soon as the intensity reaches W, any medium becomes prone to ionization-induced damage. Thus, in this regime the conventional optical elements are inappropriate. In contrast, being already ionized, plasmas are susceptible to ionization-induced damage and therefore possess infinite immunity to field-induced damage. When a laser pulse with a non-uniform spatial amplitude structure propagates through plasma, the ponderomotive force originating as a result of the intensity gradient over its cross-section results in the evacuation of plasma electrons from high-intensity regions of the irradiated regions of plasma to the low-intensity regions. The resulting redistribution of electron density makes the index of refraction of plasma intensity dependent. This intensity dependence of the index of refraction of plasma is responsible for the self-focusing and self-phase modulation (SPM) of the laser pulse. In the case of SPM, the intensity dependence of the index of refraction leads to a frequency chirp and consequently compresses the pulse to ultrashort duration [13–16]. In a physical sense, the term ultrashort characterizes a number of processes with evolution time scales that are shorter than the relaxation time for the medium.

Most laser pulses have a Gaussian irradiance profile, although it can be beneficial to use a non-Gaussian laser pulse in certain applications. Gaussian laser profiles have several disadvantages, such as the low-intensity portions on either side of the usable central region of the beam, known as "wings." These wings typically contain energy that is wasted because it is at a lower intensity than the threshold required for the given application, whether it is materials processing, laser surgery, laser-driven fusion or another application where an intensity above a given value is needed. In this regard a new class of laser pulses, known as q-Gaussian laser pulses, have attracted significant interest among researchers [17, 18]. These laser pulses are characterized by the expanded wings [19] of the irradiance profile and thus their wings contain a significant amount of energy. For the same spot size as that of an otherwise identical Gaussian laser pulse, the q-Gaussian laser pulses possess larger root mean square beam width and thus undergo less diffraction divergence. This makes q-Gaussian laser pulses superior to Gaussian laser pulses for the applications where diffraction divergence is a serious nuisance. Because no experimental or theoretical investigation on the self-compression of elliptical q-Gaussian laser pulses in collisionless plasmas has been reported to date, this gave us a strong motivation to investigate the same. Thus, this paper is aimed at presenting the first theoretical investigation of the self-compression of elliptical q-Gaussian laser pulse in collisionless plasmas with axial density ramp.

Evolution of pulse envelope. Consider the propagation of a laser pulse with an electric field vector:

$$\mathbf{E}(\mathbf{r},t) = A_0(x,y,z,t)e^{-i(k_0 z-\omega_0 t)}\, e_x ,$$

through a plasma whose equilibrium electron density is an increasing function of distance of propagation and is modeled as:

$$n_0(z) = n_0 e^{dz},$$

where (k_0, ω_0) are the vacuum wave number and angular frequency of the laser beam respectively, n_0 is the electron density at $z = 0$, and the constant d is associated with the rate of increase of electron density with distance and hence is termed the slope of the density ramp. Owing to the amplitude gradient over the cross section of the laser beam the plasma electrons experience a ponderomotive force given by:

$$F_P = -\frac{e^2}{4m\omega_0^2}\nabla\left(A_0 A_0^\star\right).$$

Here, e, m are the electronic charge and mass. As this ponderomotive force is proportional to the negative of the intensity gradient of the laser beam, it causes the evacuation of electrons from high-intensity regions of the illuminated portion of plasma. These migrated electrons accumulate in the low-intensity regions of the illuminated portion of plasma. The modified electron density of the plasma is given by:

$$n = n_0(z)\exp\left[-\frac{e^2}{8m\omega_0^2 T_0 K_0} A_0 A_0^\star\right], \quad (1)$$

where T_0 is the temperature of the plasma and K_0 is the Boltzmann constant. This modified electron density in turn alters the dielectric properties (described by the function $\varepsilon = 1 - (4\pi e^2 n)/(m\omega_0^2)$) of plasma as:

$$\varepsilon = 1 - \frac{\omega_{p0}^2}{\omega_0^2}\exp\left[-\frac{e^2}{8m\omega_0^2 T_0 K_0} A_0 A_0^\star + dz\right], \quad (2)$$

where $\omega_{p0}^2 = \frac{4\pi e^2}{m} n_0$ is the unperturbed plasma frequency, i.e., the plasma frequency in the absence of a laser beam.

Thus, the ponderomotive force on the plasma electrons produced by the laser pulse makes the index of refraction of plasma intensity dependent, which in turn, owing to the spatial dependence of the amplitude structure of the laser beam, resembles that of graded index fiber. Separating the dielectric function of plasma into linear ε_0 and nonlinear ϕ parts as:

$$\varepsilon = \varepsilon_0 + \phi\left(A_0 A_0^\star\right), \quad (3)$$

we get

$$\varepsilon_0 = 1 - \omega_{p0}^2 / \omega_0^2 , \tag{4}$$

and

$$\phi\left(A_0 A_0^\star\right) = \omega_{p0}^2 / \omega_0^2 \left\{1 - \exp\left[-\frac{e^2}{8m\omega_0^2 T_0 K_0} A_0 A_0^\star + dz\right]\right\} . \tag{5}$$

The propagation of a laser pulse through a nonlinear medium characterized by nonlinear dielectric function $\phi(A_0 A_0^*)$ is governed by wave equation:

$$i\left(\frac{\partial A_0}{\partial z} + \frac{1}{v_G}\frac{\partial A_0}{\partial t}\right) = \frac{1}{2k_0}\nabla_\perp^2 A_0 + \left(\frac{d^2 k_0}{d\omega_0^2}\right)^2 \frac{\partial^2 A_0}{\partial t^2} + \frac{k_0}{2\varepsilon_0}\phi\left(A_0 A_0^\star\right) A_0 .$$

Introducing the conventional transformation to the moving frame of the laser pulse via $(z,t) \to (z, \tau = t - z/v_G)$, we get:

$$i\frac{\partial A_0}{\partial z} = \frac{1}{2k_0}\nabla_\perp^2 A_0 + \frac{1-\varepsilon_0}{\varepsilon_0 c^2 k_0}\frac{\partial^2 A_0}{\partial \tau^2} + \frac{k_0}{2\varepsilon_0}\phi\left(A_0 A_0^\star\right) A_0 . \tag{6}$$

In the present investigation we have used a semi-analytical technique known as the variational method [20, 21] to obtain the solution to Eq. (6). This method converts the problem of solving a partial differential equation to that of solving a set of coupled ordinary differential equations. These ordinary differential equations govern the evolution of the various parameters of interest. In the case of self-focusing and self-compression of the laser pulse the parameters of interest are the beam widths and pulse width of the laser beam. The essence of method consists in finding solutions to this class of functions $A_0(r,\sigma)$, where the set of parameters $\sigma = (f_x(z), f_y(z), g(z))$ depends on the evolution variable and is determined based on the solutions of the corresponding system of ordinary differential equations. According to this method, Eq. (6) is a variational problem for the action principle based on Lagrangian density:

$$\mathcal{L} = i\left(A_0\frac{\partial A_0^\star}{\partial z} - A_0^\star\frac{\partial A_0}{\partial z}\right) + \left|\nabla_\perp A_0\right|^2 - \frac{\omega_0^2}{c^2}\int^{A_0 A_0^\star} A_0 A_0^\star \phi\left(A_0 A_0^\star\right) d\left(A_0 A_0^\star\right) + \frac{1+\varepsilon_0}{\varepsilon_0 c^2}\left|\frac{\partial A_0}{\partial \tau}\right|^2 . \tag{7}$$

In the present investigation we have considered the trial function of the form:

$$A_0\left(x,y,z,t\right) = \frac{E_{00}}{\sqrt{f_x f_y g}}\left\{1 + \frac{1}{q}\left(\frac{x^2}{a^2 f_x^2} + \frac{y^2}{b^2 f_y^2}\right)\right\}^{-q/2} \exp\left[-\frac{\left(t - z/v_G\right)^2}{2\tau_0^2 g^2}\right] . \tag{8}$$

Here, E_{00} is the axial amplitude of the field of the laser beam and a, b are the widths of the laser beam in x, y directions respectively. τ_0 is the pulse width and v_G is the group velocity of the laser pulse. The phenomenological parameter q is related to the deviation of the amplitude structure from the ideal Gaussian profile and is termed deviation parameter. The value of deviation parameter q varies from laser to laser and can be obtained by fitting into the experimental data for a given laser system. f_x, f_y, and g are the currently undetermined, real functions of only the longitudinal coordinate z. Upon multiplication with a and b respectively, f_x and f_y give the instantaneous beam widths of the laser pulse in x and y directions respectively and upon multiplication with τ_0, the function g gives the instantaneous pulse width of the laser pulse. Thus, f_x and f_y are termed dimensionless beam width parameters and g is called pulse width parameter.

Substituting the trial function given by Eq. (8) in Lagrangian density and integrating over the entire cross section of the laser beam we get the reduced Lagrangian as $L = \iint \mathcal{L} d^2 r d\tau$. The corresponding Euler–Lagrange equations:

$$\frac{d}{dz}\left(\frac{\partial L}{\partial\left(\partial\sigma/\partial z\right)}\right) - \frac{\partial L}{\partial \sigma} = 0;\ \ \sigma = f_x, f_y, g , \tag{9}$$

give

$$\frac{d^2 f_x}{d\xi^2} = \frac{\left(1 - 1/q\right)\left(1 - 2/q\right)}{\left(1 + 1/q\right)}\frac{1}{f_x^3} - \left(1 - \frac{1}{q}\right)\left(1 - \frac{2}{q}\right)\left(\frac{\omega_{p0}^2 a^2}{c^2}\right) e^{d'\xi}\frac{\beta E_{00}^2}{f_x^2 f_y g} I , \tag{10}$$

$$\frac{d^2 f_y}{d\xi^2} = \left(\frac{a}{b}\right)^4 \frac{(1-1/q)(1-2/q)}{(1+1/q)} \frac{1}{f_y^3} - \left(\frac{a}{b}\right)^2 \left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left(\frac{\omega_{p0}^2 a^2}{c^2}\right) e^{d'\xi} \frac{\beta E_{00}^2}{f_x f_y^2 g} I\ , \qquad (11)$$

$$\frac{d^2 g}{d\xi^2} = \left(\frac{a^2}{c^2 \omega_0^2 \tau_0^2}\right)^2 \frac{1}{g^3} - \left(\frac{a^2}{c^2 \omega_0^2 \tau_0^2}\right)\left(\frac{\omega_{p0}^2 a^2}{c^2}\right) e^{d'\xi} \frac{\beta E_{00}^2}{f_x f_y g^2} I, \qquad (12)$$

where $\beta = \dfrac{e^2}{8m\omega_0^2 T_0 K_0}$, $d' = dk_0 a^2$, $\xi = \dfrac{z}{k_0 a^2}$, and $t' = \dfrac{\tau}{\tau_0 g}$, $u = \left(\dfrac{x^2}{a^2 f_x^2} + \dfrac{y^2}{b^2 f_y^2}\right)^{1/2}$.

Equations (10)–(12) are the nonlinearly coupled differential equations governing the evolution of beam widths and pulse width of the elliptical q-Gaussian laser pulse during its journey through the plasma. The first terms on the right-hand sides of these equations correspond to the linear propagation of the laser pulse, i.e., its propagation through the vacuum or through the media whose index of refraction is independent of the intensity of the laser pulse. The second terms on the right-hand side (RHS) of these equations correspond to the nonlinear response of the medium. It can be seen that although in linear media the beam widths along the two transverse directions and pulse width of the laser pulse evolve independently, in the case of plasma due to the laser-induced optical nonlinearity, they become coupled to each other, i.e., in nonlinear media temporal characteristics of the laser pulse affect its spatial characteristics and vice versa.

Results and discussion. In the present investigation the fourth-order Runge–Kutta method has been used to solve Eqs. (10)–(12) numerically for the following set of laser-plasma parameters: a = 10μm, $\omega_0 = 1.78\times10^{15}$ rad/s, $\beta E_{00}^2 = 3$, $(\omega_{p0}^2 a^2)/c^2 = 6$, $\tau_0 = 10^{-15}$ s, $d' = 0.025$, and $q = (3, 4, \infty)$ and $a/b = (1, 1.1, 1.2)$.

In solving Eqs. (10)–(12) it has been assumed that at the plane of incidence, the laser pulse has plane wavefront. Mathematically, this condition means that at $\xi = 0$:

$$f_{x,y} = g = 1,$$

$$df_{x,y}/d\xi = dg/d\xi = 0.$$

Figure 1 illustrates the evolution of pulse width and beam widths of the laser pulse with longitudinal distance through the plasma. It is can be seen that the pulse width g of the laser pulse decreases monotonically

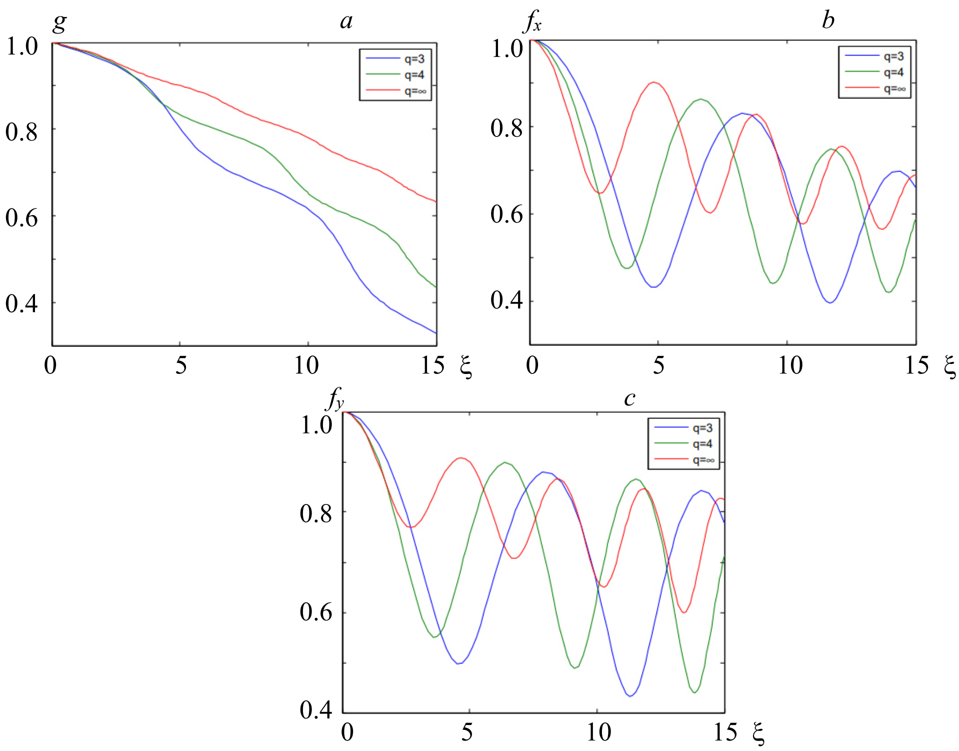

Fig. 1. Evolution of pulse width g (a) and beam widths f_x (b) and f_y (c) with distance of propagation for different values of deviation parameter q = (3, 4, ∞) and at fixed values of ellipticity a/b = 1.1.

with a distance of propagation showing step-like behavior, whereas the beam widths $f_{x,y}$ along both the transverse directions show oscillatory behavior. Each step of the pulse width is positioned at the location of the minimum beam width. The monotonic decrease in pulse width is due to the nonlinear dependence of the index of refraction of plasma on the intensity of the laser pulse. The resulting nonlinear dependence of the axial phase velocity induces a frequency chirp. From the Fourier transform of the pulse envelope from the time domain to the frequency domain, it can be seen that a pulse of finite duration contains a spread of frequencies (Fig. 2). This fact can be verified from the energy-time uncertainty, i.e., $\Delta\nu\Delta t$ = const. In order to keep the product $\Delta\nu\Delta t$ =const, a pulse with duration contains a spread of multiple frequencies. Thus, a finite pulse can be represented by a jumble of multiple frequencies. As plasma has anomalous dispersion properties, the higher frequencies (back of the pulse), move faster than the lower frequencies (front of the pulse). As a result, the pulse becomes compressed with a distance of propagation (Fig. 3). The step-like behavior of the pulse corresponding to a minimum beam width is due to the fact that the ponderomotive nonlinearity couples the beam width of the pulse with its pulse width. As the focal regions of the laser pulse are the regions of highest intensity, the maximum anomalous dispersion of the laser pulse occurs there. Hence, the pulse experiences maximum compression at its focal regions. This gives the pulse width a step-like behavior. From Fig. 1a it seems that the pulse width of the laser pulse may decrease down to zero. This is one of the limitations of the proposed model. In actuality, as the pulse width decreases below the intensity threshold of other instabilities such as stimulated Raman and Brillouin scattering, the pulse will not become compressed further and will become saturated.

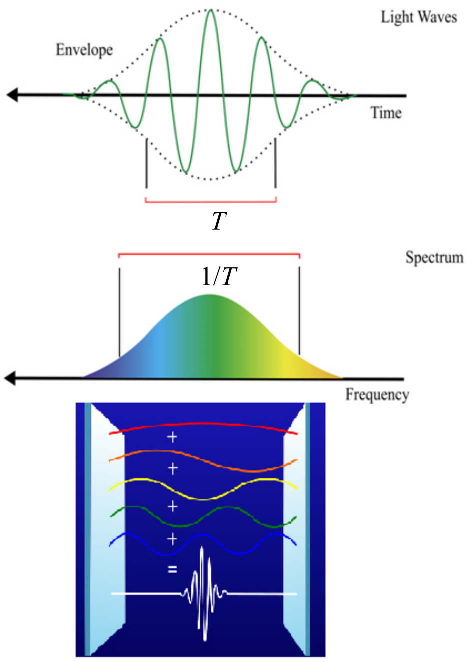

Fig. 2. Pulse envelope.

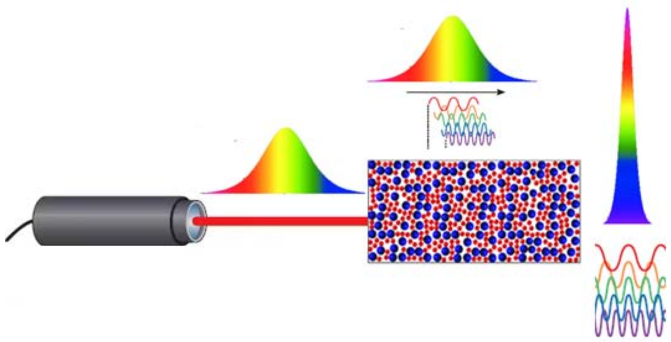

Fig. 3. Pulse compression in plasma.

The oscillatory behavior of the beam widths can be explained by analyzing the role and origin of various terms contained in the evolution equations for the beam widths, i.e., Eqs. (10) and (11). The first terms on the RHS of these equations are the spatial dispersive terms that model the spreading of the laser pulse in transverse x and y directions as a consequence of the diffraction divergence. Hence, these terms are termed as diffraction terms. The second terms on the RHS of these equations that have complex dependence on beam widths $f_{x,y}$ originate as a consequence of ponderomotive force exerted by the laser pulse on plasma electrons. These terms model the nonlinear refraction of the laser pulse and the nonlinear coupling of the beam widths along transverse directions. As a result of the laser-induced optical nonlinearity of the plasma, the resulting nonlinear refraction of the laser pulse tends to counter balance the effect of diffraction along both the transverse directions. Thus, during the propagation of laser pulse through the plasma, a competition starts between the two phenomena of diffraction and nonlinear refraction. Whether the beam widths of the laser pulse will converge or diverge is decided by the winning phenomenon. Thus, there exists a critical value of intensity (that can be obtained by balancing the two terms on the RHS of Eqs. (10) and (11)) above which the pulse converges along both the transverse directions and otherwise it will diverge spatially. In the present investigation we have kept the initial intensity of the laser pulse greater than the critical intensity. That is the reason why the beam widths of the laser pulse along both the transverse directions converge initially. As the laser pulse shrinks spatially, its intensity increases. When the intensity of the laser pulse becomes too high, the illuminated portion of the plasma is almost evacuated from the electrons. Hence, the pulse now propagates as if it is propagating through a vacuum. As a laser pulse propagating through a vacuum undergoes diffraction, the beam width of a laser pulse propagating through plasma, after attaining a possible minimum value, bounces back toward its original value. As the widths of the laser pulse along both the transverse directions start expanding, the competition between diffraction broadening and nonlinear refraction starts again. Now, this competition lasts until $f_{x,y}$ obtain their maximum possible values. These processes continue to repeat themselves and thus give oscillatory behavior to the beam widths of the laser pulse along the two transverse directions.

Further, it can be seen that after every focal spot the maximum, as well as the minimum of the beam width, shifts downward, i.e., the next focal spot of the pulse is more intense than the previous one. This is the actual motivation behind taking the density profile of the plasma in the shape of a ramp. As the equilibrium electron density of the plasma is a monotonically increasing function of distance, the plasma index of refraction continues to decrease with the penetration of the laser pulse into the plasma. Consequently, the self-focusing effect is enhanced and the maximum, as well as a minimum of the beam width, continues to shift downward after every focal spot. Another factor contributing to the enhancement of self-focusing of the laser pulse is self-compression of the laser pulse. As the laser pulse is compressed with the distance of propagation, its peak intensity increases. This in turn leads to the enhancement of its self-focusing.

The plots in Fig. 1 also indicate that with an increase in the value of deviation parameter q, the extent of self-focusing along both the transverse directions is reduced. This is because for a laser pulse with a larger value of q, most of the pulse energy is concentrated in a narrow region around the beam axis. Hence, these laser pulses receive a small contribution from the off-axial rays toward the nonlinear refraction. As the phenomenon of self-focusing is a homeostasis of nonlinear refraction of the laser pulse due to the optical nonlinearity of the medium, an increase in the value of deviation parameter q reduces the extent of self-focusing of the laser pulse. Thus, compared with q-Gaussian laser pulses, ideal Gaussian pulses possess minimum focusing character.

It can also be seen that instead of their reduced focusing, laser pulses with higher values of deviation parameter q possess faster focusing along both the transverse directions. This is due to the faster focusing character of the rays closer to the axis of the laser pulse. Being away from the axis of the pulse, off-axial rays take long to self-focus. As there are more off-axial rays in laser pulses with lower values of deviation parameter q, these laser pulses possess a slower focusing character.

The first plot in Fig. 1 indicates that with an increase in deviation parameter q, the extent of self-compression of the laser pulse is reduced. This is because, owing to the optical nonlinearity of plasma, the pulse width is coupled with the beam width. Thus, there is a one-to-one correspondence between the extent of self-focusing and self-compression. As with an increase in deviation parameter, the extent of self-focusing decreases, and there is a corresponding reduction in self-compression of the laser pulse.

Figure 4 illustrates the effect of the ellipticity of the laser pulse along the direction of the laser pulse on its self-compression and self-focusing. It can be seen that with an increase in the pulse ellipticity along the direction, there is a reduction in the extent of self-focusing of the laser pulse along the direction. This is because, at a fixed value of a, an increase in the ellipticity of the laser pulse (i.e., a/b) indicates the reduction in the initial beam width of the laser pulse along the y direction. Hence, an increase in the ellipticity of the laser pulse along the y direction makes the diffraction effect stronger along the y direction. This results in the reduced focusing of the laser pulse along the y direction.

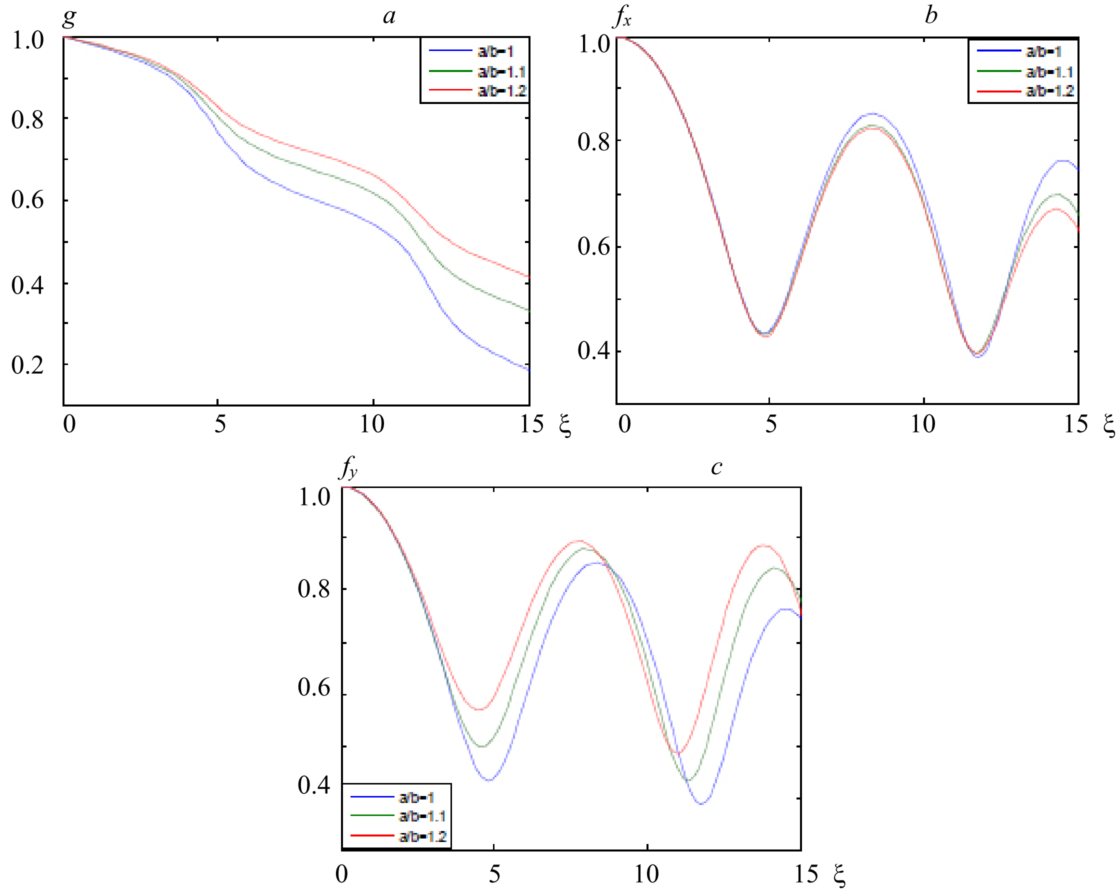

Fig. 4. Evolution of pulse width g (a) and beam widths f_x (b) and f_y (c) with distance of propagation for different values of ellipticity $a/b = (1, 1.1, 1.2)$, and at fixed values of deviation parameter $q = 3$.

It can also be seen that initially, the increase in the ellipticity of the laser pulse does not produce any significant effect on its self-focusing along the x direction. However, as the pulse penetrates deeper into the plasma the focus along the x direction also decreases. This is because, as the pulse penetrates deeper and deeper into the plasma, the nonlinear coupling between the two beam widths becomes stronger and stronger.

The first plot in Fig. 4 indicates that with an increase in the ellipticity of the laser pulse, the extent of its self-compression decreases. This is because, owing to the optical nonlinearity of the plasma, the pulse width of the laser pulse is coupled with its beam width. As with the increase in ellipticity, the overall extent of self-focusing of the laser pulse decreases, which in turn leads to a reduction in the extent of self-compression.

Conclusions. Self-compression of elliptical q-Gaussian laser pulses propagating through axially inhomogeneous plasmas has been investigated. The effect of self-focusing of the laser pulse on self-compression has been incorporated. From the results of the present investigation, it can be concluded that there is a one-to-one correspondence between the extent of self-focusing and self-compression of the laser pulse. As the spatial amplitude structure of the laser pulse converges toward the ideal Gaussian distribution, the extent of self-compression of the laser pulse decreases.

There are certain limitations of the proposed model. As the intensity of the laser pulse increases owing to the combined effects of self-focusing and self-compression, other parametric instabilities such as stimulated Raman and Brillouin scattering, filamentation of the pulse, etc., will come into the picture. The effect of these nonlinear phenomena has not been incorporated into the proposed model, as it would increase the mathematical complexity of the model.

The results of the present investigation may serve as a guide for the experimentalists working in the area of laser–plasma interactions.

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