PRECISE WAVELENGTHS OF $4p(^2P_{1/2})\rightarrow nd\ ^2D_{3/2}$ and $4p(^2P_{3/2})\rightarrow nd\ ^2D_{3/2,5/2}$ RYDBERG TRANSITIONS IN NEUTRAL POTASSIUM CALCULATED VIA THE SCREENING CONSTANT PER UNIT NUCLEAR CHARGE METHOD[**]

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Photoionization of neutral potassium K I is investigated in the framework of the screening constant per unit nuclear charge (SCUNC) method. Transition energies and wavelengths belonging to $4p(^2P_{1/2})\rightarrow nd\ ^2D_{3/2}$ and $4p(^2P_{3/2})\rightarrow nd\ ^2D_{3/2,5/2}$ Rydberg transitions are reported. Accurate transition energies and wavelengths originating from $4p(^2P_{1/2,3/2})$ levels of K I are tabulated for $20 \le n \le 100$. The SCUNC wavelengths are believed to be the first calculations that agree excellently with the existing experimental measurements up to n = 70 using linearly polarized laser light. The maximum shift in wavelengths relative to the experimental data is at 0.03 nm up to n = 70. New wavelengths are tabulated for n = 71–100 along with new transition energies for n = 20–100.

Keywords: *photoionization, transition energies, wavelengths, Rydberg transition, neutral K I, screening constant per unit nuclear charge.*

ТОЧНЫЕ ДЛИНЫ ВОЛН РИДБЕРГОВСКИХ ПЕРЕХОДОВ $4p(^2P_{1/2})\rightarrow nd\ ^2D_{3/2}$ и $4p(^2P_{3/2})\rightarrow nd\ ^2D_{3/2,5/2}$ В НЕЙТРАЛЬНОМ АТОМЕ КАЛИЯ, РАССЧИТАННЫЕ ЧЕРЕЗ КОНСТАНТЫ ЭКРАНИРОВАНИЯ НА ЕДИНИЦУ ЯДЕРНОГО ЗАРЯДА

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Фотоионизация нейтрального атома калия K I исследована в рамках формализма константы экранирования на единицу заряда ядра (SCUNC). Рассчитаны энергии и длины волн ридберговских переходов $4p(^2P_{1/2})\rightarrow nd\ ^2D_{3/2}$ и $4p(^2P_{3/2})\rightarrow nd\ ^2D_{3/2,5/2}$ для $20 \le n \le 100$. Длины волн SCUNC являются первыми расчетами, которые хорошо согласуются с экспериментальными измерениями до n = 70 с использованием линейно поляризованного лазерного излучения. Максимальный сдвиг длин волн относительно экспериментальных данных составляет 0.03 нм до n = 70. Получены значения длин волн для n = 71–100 и энергий переходов для n = 20–100.

Ключевые слова: *фотоионизация, энергия переходов, длина волны, ридберговский переход, нейтральный калий K I, константа экранирования на единицу заряда ядра.*

Introduction. Photoionization is a fundamental process playing a key role in plasma ions–light interaction. Potassium is the seventh most abundant element, therefore, the photoionization studies of neutral potassium K I are very challenging. Several theoretical and experimental methods have been exploited in the recent past to study the photoionization of K I. On the theoretical side, the photoionization cross-sections for Na I, K I, Rb I, and Cs I were investigated by Savukov [1] who applied the relativistic many-body perturba-

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tion theory by the use of quasi-continuum B-spline orbitals. In addition, the photoionization cross-sections from the ground state and 4*p*, 5*s*–7*s*, and 3*d*–5*d* excited states of K I were investigated by Zatsarinny and Tayal [2] in the framework of the Dirac-based B-spline R matrix method. On the experimental side, Amin et al. [3] measured the photoionization cross-sections from $4p^2P_{3/2}$ and $4p^2P_{1/2}$ excited states of K I at 355 nm using a Nd:YAG laser in conjunction with a thermionic diode ion detector. Yar et al. [4, 5] reported on the photoionization cross-section from 6p- excited state of K I at three ionizing laser wavelengths. Following their investigations, Kalyar et al. [6] used a two-step ionization/excitation technique to measure the photoionization cross-sections from $4p^2P_{3/2,1/2}$ excited states and oscillator strengths measurements for $4p^2P_{1/2,3/2}$–$nd\ ^2D_{3/2,5/2}$ Rydberg states of K I. As is shown in the work of Noll et al. [7], the use of sunlight or a laser source permits detection of the presence of K I in the Earth's mesopause region via its resonance fluorescence K (D1) line located at 769.9 nm. As far as astrophysical systems are concerned, the potassium excess absorption around 7699 Å of HD189733b and HD209458b exoplanets have been studied experimentally by Keles et al. [8]. In addition, Allard et al. [9] indicated that among the most alkali atoms, Na I and K I are due to the important role of opacity in the atmospheres of the brown dwarfs and exoplanets. In the same way, of great interest in detecting dark matter are H-like potassium (K XIX) X-ray spectrum and its He-like (K XVIII) satellite lines located between 3.34 and 3.39 Å [10]. The previous lines were measured by Weller et al. [10] using a long-pulse (2 ns) beam at the Orion laser facility. Very recently, photoionization and photon-atom scattering cross-sections for Li I, Na I, K I, Rb I, and Cs I were investigated by Singor et al. [11] who used a fully relativistic approach. In the work of Kalyar et al. [6], previously mentioned, the high-lying $4p(^2P_{1/2})\rightarrow nd\ ^2D_{3/2}$ and $4p(^2P_{3/2})\rightarrow nd\ ^2D_{3/2,5/2}$ Rydberg transitions have been reported for $20 \le n \le 70$. However, to the best of our knowledge, no theoretical studies have been performed to compare with the experimental data of Kalyar et al. [6]. The main motivation of the present work is to report accurate transition energies and wavelengths belonging to $4p(^2P_{1/2})\rightarrow nd\ ^2D_{3/2}$ and $4p(^2P_{3/2})\rightarrow nd\ ^2D_{3/2,5/2}$ Rydberg transitions to be compared with the available experimental data [6]. Tabulation of new theoretical wavelengths for $71 \le n \le 100$ and new transition energies for $n = 20$–100 is also a goal of this study. For this purpose, we apply the screening constant per unit nuclear charge (SCUNC) formalism as being very suitable in the study of the photoionization of atomic systems [12–19]. Recently, the SCUNC formalism has been used to report accurate wavelengths for three of the most intense lines (resonance line: $1s^2\ ^1S_0$–$1s2p\ ^1P_1$, intercombination line: $1s^2\ ^1S_0$–$1s2p\ ^3P_1$, and forbidden line: $1s^2\ ^1S_0$–$1s2s\ ^3S_1$) and wavelengths for $1s^2\ ^1S_0\rightarrow 1snp\ ^1P_1$ and $1s^2\ ^1S_0\rightarrow 1snp^3P_2$ ($2 \le n \le 25$) transitions in He-like systems ($Z = 2$–13) [17]. The present work is the second study testing the suitability of the SCUNC formalism to report accurate wavelengths for a more complex atomic system such as K I. The paper is organized as follows–section 2 presents a summary of SCUNC formalism, section 3 presents a discussion of the results obtained compared to the measurements of Kalyar et al. [6], and section 4 summarizes and concludes the present study.

Theory. The starting point of the SCUNC formalism is the total energy of $(Nl, nl')^{2S+1}L^{\pi}$ excited states of He-like systems (in Rydberg), given as [12–19]:

$$E = -Z^2\left(1/N^2 + 1/n^2\left[1 - \beta\left(Nl, nl'; {}^{2S+1}L^{\pi}; Z\right)\right]^2\right). \quad (1)$$

In this equation, the principal quantum numbers N and n correspond to the inner and the outer electron of the helium-isoelectronic series. The β-parameters are screening constants by unit nuclear charge expanded in inverse powers of Z and given by

$$\beta\left(Nl\,nl'; {}^{2S+1}L^{\pi}; Z\right) = \sum_{k=1}^{q} f_k \left(1/Z\right)^k, \quad (2)$$

where $f_k = f_k(Nl, nl'; {}^{2S+1}L^{\pi})$ are the parameters to be evaluated empirically.

For a given Rydberg series originating from a $^{2S+1}L_J$ state, we obtain

$$E_n = E_\infty - \frac{Z^2}{n^2}\left[1 - \beta(nl; s, \mu, \nu, {}^{2S+1}L^{\pi}Z)\right]^2, \quad (3)$$

where ν and μ (μ > ν) are the principal quantum numbers of $(^{2S+1}L_J)\ \nu l$ ($n = >\nu$) and $(^{2S+1}L_J)\ \mu l$ ($n = \mu$) Rydberg series used in the empirical determination of f_k-screening constants, s is the spin of nl-electron ($s = 1/2$), E_∞ is the energy value of the series limit, E_n is the resonance energy, and Z is the atomic number. β-parameters are SCUNC expanded in inverse powers of Z and given by

$$\beta\left(Z, {}^{2S+1}L_J, n, s, \mu, \nu\right) = \sum_{k=1}^{q} f_k \left(\frac{1}{Z}\right)^k, \quad (4)$$

where $f_k = f_k(^{2S+1}L_J, n, s, \mu, \nu)$ are screening constants to be evaluated empirically. In Eq. (2), q stands for the number of terms in the expansion of the β-parameter. The resonance energies are in the form

$$E_n = E_\infty - \frac{Z^2}{n^2}\left\{1 - \frac{f_1(^{2S+1}L_J^\pi)}{Z(n-1)} - \frac{f_2(^{2S+1}L_J^\pi)}{Z} \pm \sum_{k=1}^{q}\sum_{k'=1}^{q'} f_1^{k'} F(n,\mu,\nu,s)\left(1/Z\right)^k\right\}^2, \quad (5)$$

where $\pm\sum_{k=1}^{q}\sum_{k'=1}^{q'} f_1^{k'} F(n,\mu,\nu,s)\left(1/Z\right)^k$ corrective term is introduced to stabilize the resonance energies by increasing the principal quantum number n. In general, resonance energies are analyzed from the standard quantum-defect expansion formula as:

$$E_n = E_\infty - \frac{R\,Z_{\text{core}}^2}{(n-\delta)^2}, \quad (6)$$

where R is the Rydberg constant, E_∞ is the ionization potential, Z_{core} is the electric charge of the core ion, and δ is the quantum defect. In addition, theoretical and measured energy positions can be analyzed by calculating Z^*-effective charge in the framework of the SCUNC procedure;

$$E_n = E_\infty - RZ^{*2}/n^2 \quad (7)$$

Besides, comparing Eqs. (5) and (7), the effective nuclear charge is in the form

$$Z^* = Z\left\{1 - \frac{f_1(^{2S+1}L^\pi)}{Z(n-1)} - \frac{f_2(^{2S+1}L^\pi)}{Z} \pm \sum_{k=1}^{q}\sum_{k'=1}^{q'} f_1^{k'} F(n,\mu,\nu,s)\left(1/Z\right)^k\right\}. \quad (8)$$

The f_2-parameter in Eq. (2) can be determined theoretically from Eq. (8) by neglecting the corrective term with the condition:

$$\lim_{n\to\infty} Z^* = Z\left(1 - \frac{f_2(^{2S+1}L^\pi)}{Z}\right) = Z_{\text{core}}. \quad (9)$$

Thus:

$$f_2 = Z - Z_{\text{core}}. \quad (10)$$

The photoionization process from an atomic X^{p+} system is given by

$$X^{p+} + h\nu \rightarrow X^{(p+1)+} + e^-. \quad (11)$$

Using [11], we find $Z_{\text{core}} = p + 1$.

As it is shown in the work of Kalyar et al. [6], photoionization from $4p^2P_{1/2,3/2}$ excited states of K I results in a potassium ion and an outgoing electron into the associated continuum channels. Two processes leading to $\varepsilon d\,^2D_{3/2,5/2}$ continuum channels are the following:

$$\text{K}\,(4p\,^2P_{1/2}) + h\nu \rightarrow \varepsilon s\,^2S_{1/2} + \varepsilon d\,^2D_{3/2}, \quad (12)$$

$$\text{K}\,(4p\,^2P_{3/2}) + h\nu \rightarrow \varepsilon s\,^2S_{1/2} + \varepsilon d\,^2D_{3/2,5/2}. \quad (13)$$

In these two processes, $\varepsilon\,\ell$ denotes the continuum channels relative to the s ($\ell = 0$) state where the total quantum number $J = 1/2$ or to d ($\ell = 2$) states with $J = 3/2, 5/2$ in the framework of the LS coupling scheme $J = \ell \pm s = \ell \pm 1/2$ (here, s denotes the electron spin). It should be mentioned that the strongest series accessed from $4p^2P_{3/2}$ intermediate level are $nd\,^2D_{3/2,5/2}$ and the other relatively weaker series is $ns\,^2S_{1/2}$. However, the Rydberg series attached with $4p^2P_{1/2}$ intermediate level are assigned as $nd\,^2D_{3/2}$ and $ns\,^2S_{1/2}$ [6].

In the experiments of Kalyar et al. [6], two dye lasers that were simultaneously pumped by a common Nd: YAG laser were used. The first dye laser was charged with LD-751 dye dissolved in methanol and pumped by the SHG (532 nm) of the Nd:YAG laser, to excite atoms from the ground state to $4p^2P_{3/2}$ by tuning it at 766.7 nm or to $4p\,^2P_{1/2}$ by tuning it at 770.1nm.

In order to measure the photoionization cross-sections, the atoms from $4s\,^2S_{1/2}$ ground level were promoted either to $4p^2P_{1/2}$ or $4p^2P_{3/2}$ using a dye laser tuned at 12985.17 or 13042.89 cm^{-1}, respectively. According to the selection rules, the magnetic quantum number m_J holds the values $-J$ up to $+J$ and takes then $(2J + 1)$ values. So, for the $4s\,^2S_{1/2}$ ground level, $m_J = \pm 1/2$, and for the $4p$ excited level, $m_J = \pm 3/2$ and $\pm 1/2$. It should be emphasized that, due to the linearly polarized laser light applied in the experiments [6], only transitions relative to $m_J = \pm 1/2$ states are allowed following the selection rule $\Delta m_J = \pm 0$. In the second step, the atoms from either the $^2P_{1/2}$ or $^2P_{3/2}$ intermediate level were ionized by the second laser to explore the specific energy region of the continuum.

A two-step excitation technique was used to record the Rydberg transitions associated with the $4p^2P_{1/2,3/2}$ levels to extract the oscillator strengths of the $4p^2P_{1/2,3/2}\rightarrow nd\,^2D_{3/2,5/2}$ Rydberg series. Initially, atoms from the ground state were resonantly excited to the $4p^2P_{3/2}$ level. In the second step, the dye laser was scanned covering the photon energy region from 21.682 to 22.682 cm^{-1} to record $4p^2P_{3/2}\rightarrow nd\,^2D_{3/2,5/2}$ ($20 \le n \le$ limit) Rydberg series. The term energies were obtained by adding the energies of the exciting dye laser to the energies of the $4p^2P_{1/2}$ (12985.17 cm^{-1}) or $4p^2P_{3/2}$ (13042.89 cm^{-1}) intermediate levels [6].

Results and discussion. For the neutral potassium excitation considered in this work, Eq. (11) gives $K+h\nu\rightarrow K^+ + e^-$. Thus, for K I, $Z_{core} = 1$ and then $f_2 = (19–1) = 18.0$. The remaining f_1-parameter in Eq. (5) is determined empirically using the experimental data of Kalyar et al. [6] for μl level corresponding here to $nd\,^2D_{3/2,5/2}$ states of K I. In the present study, the resonance energies relative to Rydberg $4p(^2P_{1/2})\rightarrow nd\,^2D_{3/2}$ and $4p(^2P_{3/2})\rightarrow nd\,^2D_{3/2,5/2}$ transitions in K I are established from the following equation (in Rydberg units):

$$E_n = E_\infty - \frac{Z^2}{n^2}\left\{1 - \frac{f_1(^{2S+1}L_J^\pi)}{Z(n-1)} - \frac{f_2(^{2S+1}L_J^\pi)}{Z} + \sum_{k=1}^{q}\sum_{k'=1}^{q'} f_1^{k'} F(n,\mu,\nu,s)\left(1/Z\right)^k\right\}^2. \tag{14}$$

For $4p(^2P_{1/2})\rightarrow nd\,^2D_{3/2}$ transitions, from Eq. (14) we get

$$E_n = IP - \frac{Z^2}{n^2}\left\{1 - \frac{f_1(^2D_{3/2})}{Z(n-1)} - \frac{f_2(^2D_{3/2})}{Z} + \frac{f_1(^2D_{3/2})\times(n-\mu)^2}{Z^2(n-\mu+\ell+2s)} + \frac{f_1(^2D_{3/2})\times(n-\mu)^2}{Z^3(n-\mu+\ell)} + \right.$$
$$\left. + \frac{f_1(^2D_{3/2})\times(n-\mu)^3}{Z^4(n-\mu+\ell)} + \frac{f_1(^2D_{3/2})\times(n-\mu)^3}{Z^5(n-\mu+\ell)}\right\}^2. \tag{15}$$

For $4p(^2P_{3/2})\rightarrow nd\,^2D_{3/2,5/2}$ transitions, from Eq. (14) we obtain

$$E_n = IP - \frac{Z^2}{n^2}\left\{1 - \frac{f_1(^2D_{3/2,5/2})}{Z(n-1)} - \frac{f_2(^2D_{3/2,5/2})}{Z} + \frac{f_1(^2D_{3/2,5/2})(n-\mu)^2}{Z^2(n-\mu+2\ell+s+1)} + \right.$$
$$\left. + \frac{f_1(^2D_{3/2,5/2})\times(n-\mu)^2}{Z^3(n-\mu+2\ell+2s+1)} + \frac{f_1(^2D_{3/2,5/2})(n-\mu)^2}{Z^4(n-\mu+2\ell+2s+1)} + \frac{f_1(^2D_{3/2,5/2})(n-\mu)^2}{Z^5(n-\mu+2\ell+2s+1)}\right\}^2. \tag{16}$$

In Eqs. (15) and (16), ℓ denotes the orbital quantum number ($\ell = 2$) and s ($s = 1/2$) is the spin of the nd electron. As far as the wavelengths are concerned, they are calculated by the well-known relation $\Delta E = hc/\lambda$, using the value of Planck constant $h = 6.62606896\ (33)\times10^{-34}$ J · s, the velocity of light in vacuum $c = 299792458$ m/s, and 1 eV = 1.602179487 (40)×10^{-19} J [20]:

$$\Delta E = E_n(nd\,^2D_J) - E_n(4p^2P_J) = 1239.839554/\lambda, \tag{17}$$

where $^2D_J = {}^2D_{3/2}$ or $^2D_{3/2,\,5/2}$ respectively for $^2P_J = {}^2P_{1/2}$ or $^2P_{3/2}$.

To extract empirically $f_1(^2D_{3/2,5/2})$-screening constants in Eqs. (15) and (16) from the value of the ionization potential of K I = 35009.81 cm^{-1}, Rydberg constant for K I as 109735.78 cm^{-1}, the energies of $4p\ ^2P_{1/2}$ = 12985.17 cm^{-1} and that of $4p^2P_{3/2}$ level = 13042.89 cm^{-1} reported by Kalyar et al. [6], an energy conversion 1 cm^{-1} = 0.00012398424 eV was used. So, we obtain IP = 4.340665 eV; $E(4p^2P_{1/2})$ = = 1.6099564 eV; $E(4p^2P_{3/2})$ = 1.6171128 eV. Let us then consider the first transition in K I for each of the $4p(^2P_{1/2})\rightarrow nd\,^2D_{3/2}$ and $4p(^2P_{3/2})\rightarrow nd\,^2D_{3/2,5/2}$ Rydberg transitions. From Kalyar et al. [6], we extract the following wavelengths: for $4p(^2P_{1/2})\rightarrow 20d\,^2D_{3/2}$ (μ = 20) transition: λ = 459.79 nm, for $4p(^2P_{3/2})\rightarrow 20d\,^2D_{3/2,5/2}$ (μ = 20) transition: λ = 461.02 nm. Using these wavelengths and the values IP and $E(4p^2P_{1/2,3/2})$ we obtained, Eq (17) gives $E\ (20d\,^2D_{3/2})$ = 4.3064908 eV; $E\ (20d\,^2D_{3/2,5/2})$ = 4.3064528 eV. Using this values Eqs. (15) and (16) give respectively $f_1(^2D_{3/2})$ = –0.044755; $f_1(^2D_{3/2,5/2})$ = –0.055341. Using this results Eqs. (15) and (16) explicitly give the following formulas:

for $4p\ (^2P_{1/2})\rightarrow nd\,^2D_{3/2}$ transitions

$$E_n = 4.340665 - \frac{19^2}{n^2}\left\{1 + \frac{0.044755}{19(n-1)} - \frac{18}{19} - \frac{0.044755\times(n-20)^2}{19^2(n-17)} - \frac{0.044755\times(n-20)^2}{19^3(n-18)} - \right.$$
$$\left. - \frac{0.044755\times(n-20)^3}{19^4(n-18)} - \frac{0.044755\times(n-20)^3}{19^5(n-18)}\right\}^2. \tag{18}$$

For $4p\ (^2P_{3/2})\rightarrow nd\,^2D_{3/2,5/2}$ transitions

$$E_n = 4.340665 - \frac{19^2}{n^2}\left\{1 + \frac{0.055341}{19(n-1)} - \frac{18}{19} - \frac{0.055341\times(n-20)^2}{19^2(n-14.5)} - \frac{0.055341\times(n-20)^2}{19^3(n-14)} - \right.$$
$$\left. - \frac{0.055341\times(n-20)^2}{19^4(n-14)} - \frac{0.055341\times(n-20)^2}{19^5(n-14)}\right\}^2. \quad (19)$$

The present SCUNC calculations for both transition energies and wavelengths belonging to $4p(^2P_{1/2})\rightarrow nd\ ^2D_{3/2}$ and $4p(^2P_{3/2})\rightarrow nd\ ^2D_{3/2,5/2}$ Rydberg transitions in K I are listed in Table 1. The theoretical SCUNC wavelengths (λ_{theo}) and the experimental wavelengths (λ_{exp}) measured from the linearly polarized laser light (LPLL) experiments of Kalyar et al. [6] are presented in Table 2. A comparison shows that the SCUNC formalism excellently reproduces the experimental data. The maximum shift in wavelengths relative to the experimental data is at 0.03 nm. The excellent agreements between theory and experiment demonstrate again the suitability of the SCUNC method to report accurate photoionization data within a simple formalism up to higher values of the principal quantum number n.

TABLE 1. Screening Constant per Unit Nuclear Charge Calculations of Resonance Energies (E_n) and Wavelengths (λ) Belonging to $4p(^2P_{3/2})\rightarrow nd\ ^2D_{3/2,5/2}$ and $4p(^2P_{1/2})\rightarrow nd\ ^2D_{3/2}$ Transitions in the Potassium Atom

n	E_n, eV	λ, nm	n	E_n, eV	λ, nm
$4p(^2P_{3/2})\rightarrow nd\ ^2D_{3/2,5/2}$					
20	4.306453	461.020	61	4.337770	455.713
21	4.309672	460.469	62	4.337882	455.695
22	4.312498	459.986	63	4.337988	455.677
23	4.314983	459.562	64	4.338089	455.660
24	4.317174	459.189	65	4.338185	455.644
25	4.319113	458.860	66	4.338276	455.629
26	4.320835	458.568	67	4.338363	455.614
27	4.322369	458.308	68	4.338446	455.600
28	4.323742	458.075	69	4.338525	455.587
29	4.324974	457.867	70	4.338600	455.574
30	4.326084	457.679	71	4.338672	455.562
31	4.327086	457.510	72	4.338741	455.551
32	4.327995	457.357	73	4.338806	455.540
33	4.328820	457.217	74	4.338869	455.529
34	4.329572	457.091	75	4.338929	455.519
35	4.330260	456.975	76	4.338987	455.510
36	4.330889	456.869	77	4.339042	455.500
37	4.331466	456.772	78	4.339095	455.491
38	4.331997	456.682	79	4.339146	455.483
39	4.332486	456.600	80	4.339194	455.475
40	4.332938	456.524	81	4.339241	455.467
41	4.333356	456.454	82	4.339286	455.460
42	4.333744	456.389	83	4.339329	455.452
43	4.334104	456.328	84	4.339370	455.445
44	4.334438	456.272	85	4.339410	455.439
45	4.334750	456.220	86	4.339448	455.432
46	4.335040	456.171	87	4.339485	455.426
47	4.335312	456.125	88	4.339520	455.420
48	4.335566	456.083	89	4.339554	455.415
49	4.335803	456.043	90	4.339587	455.409
50	4.336026	456.006	91	4.339619	455.404
51	4.336235	455.970	92	4.339649	455.399
52	4.336432	455.937	93	4.339678	455.394
53	4.336617	455.906	94	4.339707	455.389

Continue Table 1

n	E_n, eV	λ, nm	n	E_n, eV	λ, nm
54	4.336792	455.877	95	4.339734	455.385
55	4.336956	455.850	96	4.339760	455.380
56	4.337111	455.824	97	4.339786	455.376
57	4.337258	455.799	98	4.339811	455.372
58	4.337396	455.776	99	4.339834	455.368
59	4.337528	455.754	100	4.339857	455.364
60	4.337652	455.733	101	4.339880	455.360
$4p(^2P_{1/2})\rightarrow nd\ ^2D_{3/2}$					
20	4.306491	459.790	61	4.337737	454.523
21	4.309714	459.241	62	4.337850	454.504
22	4.312548	458.759	63	4.337957	454.487
23	4.315036	458.338	64	4.338059	454.470
24	4.317225	457.967	65	4.338156	454.453
25	4.319159	457.640	66	4.338249	454.438
26	4.320874	457.350	67	4.338337	454.423
27	4.322401	457.093	68	4.338421	454.409
28	4.323766	456.863	69	4.338501	454.396
29	4.324990	456.657	70	4.338578	454.383
30	4.326092	456.472	71	4.338651	454.371
31	4.327088	456.305	72	4.338721	454.359
32	4.327990	456.153	73	4.338787	454.348
33	4.328810	456.016	74	4.338852	454.338
34	4.329557	455.890	75	4.338913	454.327
35	4.330239	455.776	76	4.338972	454.318
36	4.330864	455.671	77	4.339028	454.308
37	4.331438	455.575	78	4.339082	454.299
38	4.331966	455.487	79	4.339134	454.291
39	4.332453	455.405	80	4.339183	454.282
40	4.332903	455.330	81	4.339231	454.274
41	4.333319	455.261	82	4.339277	454.267
42	4.333705	455.196	83	4.339321	454.259
43	4.334064	455.136	84	4.339364	454.252
44	4.334397	455.080	85	4.339404	454.246
45	4.334708	455.028	86	4.339444	454.239
46	4.334998	454.980	87	4.339481	454.233
47	4.335270	454.935	88	4.339518	454.227
48	4.335523	454.892	89	4.339553	454.221
49	4.335761	454.853	90	4.339587	454.215
50	4.335984	454.815	91	4.339619	454.210
51	4.336194	454.780	92	4.339651	454.205
52	4.336391	454.748	93	4.339681	454.199
53	4.336577	454.717	94	4.339710	454.195
54	4.336752	454.687	95	4.339739	454.190
55	4.336917	454.660	96	4.339766	454.185
56	4.337073	454.634	97	4.339792	454.181
57	4.337221	454.609	98	4.339818	454.177
58	4.337360	454.586	99	4.339842	454.173
59	4.337493	454.564	100	4.339866	454.169
60	4.337618	454.543	101	4.339889	454.165

TABLE 2. Wavelengths (λ, nm) Belonging to $4p(^2P_{3/2})\rightarrow nd\,^2D_{3/2,5/2}$ and $4p(^2P_{1/2})\rightarrow nd\,^2D_{3/2}$ Transitions in the Potassium atom. The Theoretical Wavelengths (λ_{calc}) Calculated via the Screening Constant per Unit Nuclear Charge (SCUNC) are Compared to Experimental Wavelengths (λ_{exp}) Measured via the Linearly Polarized Laser Light (LPLL) Experiments of Kalyar et al. [6]

n	λ_{calc}^{SCUNC}	λ_{exp}^{LPLL}	$\lvert\Delta\lambda\rvert$	n	λ_{calc}^{SCUNC}	λ_{exp}^{LPLL}	$\lvert\Delta\lambda\rvert$
$4p(^2P_{3/2})\rightarrow nd\,^2D_{3/2,5/2}$							
20	461.02	461.02	0.00	46	456.17	456.19	0.02
21	460.47	460.46	0.01	47	456.13	456.14	0.01
22	459.99	459.97	0.02	48	456.08	456.10	0.02
23	459.56	459.55	0.01	49	456.04	456.06	0.02
24	459.19	459.18	0.01	50	456.01	456.02	0.01
25	458.86	458.85	0.01	51	455.97	455.99	0.02
26	458.57	458.56	0.01	52	455.94	455.95	0.01
27	458.31	458.31	0.00	53	455.91	455.92	0.01
28	458.08	458.08	0.00	54	455.88	455.89	0.01
29	457.87	457.87	0.00	55	455.85	455.86	0.01
30	457.68	457.69	0.01	56	455.82	455.83	0.01
31	457.51	457.52	0.01	57	455.80	455.81	0.01
32	457.36	457.37	0.01	58	455.78	455.78	0.00
33	457.22	457.24	0.02	59	455.75	455.76	0.01
34	457.09	457.11	0.02	60	455.73	455.74	0.01
35	456.97	456.99	0.02	61	455.71	455.72	0.01
36	456.87	456.89	0.02	62	455.69	455.70	0.01
37	456.77	456.79	0.02	63	455.68	455.68	0.00
38	456.68	456.70	0.02	64	455.66	455.66	0.00
39	456.60	456.62	0.02	65	455.64	455.64	0.00
40	456.52	456.55	0.03	66	455.63	455.63	0.00
41	456.45	456.48	0.03	67	455.61	455.61	0.00
42	456.39	456.41	0.02	68	455.60	455.60	0.00
43	456.33	456.35	0.02	69	455.59	455.59	0.00
44	456.27	456.29	0.02	70	455.57	455.57	0.00
45	456.22	456.24	0.02	71	455.56	—	—
$4p(^2P_{1/2})\rightarrow nd\,^2D_{3/2}$							
20	459.79	459.79	0.00	46	454.98	454.99	0.01
21	459.24	459.23	0.01	47	454.93	454.95	0.02
22	458.76	458.75	0.01	48	454.89	454.90	0.01
23	458.34	458.33	0.01	49	454.85	454.86	0.01
24	457.97	457.96	0.01	50	454.82	454.82	0.00
25	457.64	457.63	0.01	51	454.78	454.79	0.01
26	457.35	457.35	0.00	52	454.75	454.75	0.00
27	457.09	457.09	0.00	53	454.72	454.72	0.00
28	456.86	456.87	0.01	54	454.69	454.69	0.00
29	456.66	456.66	0.00	55	454.66	454.66	0.00
30	456.47	456.48	0.01	56	454.63	454.64	0.00
31	456.30	456.31	0.01	57	454.61	454.61	0.00
32	456.15	456.16	0.01	58	454.59	454.59	0.00
33	456.02	456.03	0.01	59	454.56	454.56	0.00
34	455.89	455.90	0.01	60	454.54	454.54	0.00
35	455.78	455.79	0.01	61	454.52	454.52	0.00
36	455.67	455.69	0.02	62	454.50	454.50	0.00
37	455.58	455.59	0.01	63	454.49	454.48	0.01
38	455.49	455.50	0.01	64	454.47	454.46	0.01
39	455.41	455.42	0.01	65	454.45	454.45	0.00
40	455.33	455.34	0.01	66	454.44	454.43	0.01

Continue Table 2

n	λ_{calc}^{SCUNC}	λ_{exp}^{LPLL}	$\lvert\Delta\lambda\rvert$	n	λ_{calc}^{SCUNC}	λ_{exp}^{LPLL}	$\lvert\Delta\lambda\rvert$
41	455.26	455.27	0.01	67	454.42	454.42	0.00
42	455.20	455.21	0.01	68	454.41	454.40	0.01
43	455.14	455.15	0.01	69	454.40	454.39	0.01
44	455.08	455.09	0.01	70	454.38	454.38	0.00
45	455.03	455.04	0.01	71	454.37	–	–

Conclusions. In the present work very precise photoionization transition energies and wavelengths belonging to $4p(^2P_{1/2})\rightarrow nd\ ^2D_{3/2}$ and $4p(^2P_{3/2})\rightarrow nd\ ^2D_{3/2,5/2}$ Rydberg transitions in K I are reported within a very soft formalism. The possibility to use the SCUNC formalism to helpfully assist the experimenters for the analysis of atomic spectra from the photoionization study is again demonstrated in this work. The new wavelengths tabulated for n = 71–100 along with the new transitions energies for n = 20–100 may be useful guidelines for the NIST (National Institute of Standards and Technology) database and future studies on the photoionization of K I as far as $4p(^2P_{1/2})\rightarrow nd\ ^2D_{3/2}$ and $4p(^2P_{3/2})\rightarrow nd\ ^2D_{3/2,5/2}$ Rydberg transitions are concerned.

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