SELF-FOCUSING OF RIPPLED ELLIPTICAL *q*-GAUSSIAN LASER BEAM IN PLASMA WITH AXIAL DENSITY RAMP

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Theoretical investigation on self-focusing of an elliptical q-Gaussian laser beam carrying an intensity ripple over its cross section, in plasma with an axial density ramp has been presented. The optical nonlinearity of plasma has been modeled by the relativistic mass nonlinearity of plasma electrons in the field of a laser beam. Using the variational theory approach, semi-analytical solutions of the wave equations for the fields of the main beam and that of the ripple have been obtained. Emphasis has been put on the evolutions of the beam widths of the main beam and that of the ripple.

Keywords: *q-Gaussian, laser ripple, variational theory, clean energy, self-focusing.*

САМОФОКУСИРОВКА ВОЗМУЩЕННОГО ЭЛЛИПТИЧЕСКОГО *q*-ГАУССОВА ЛАЗЕРНОГО ПУЧКА В НЕОДНОРОДНОЙ ПЛАЗМЕ

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УДК 621.373;533.9

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(Поступила 7 октября 2022)

Исследована самофокусировка эллиптического q-гауссова лазерного пучка с импульсами интенсивности по поперечному сечению в неоднородной плазме. Оптическая нелинейность плазмы моделировалась релятивистской массовой нелинейностью электронов плазмы в поле лазерного луча. С использованием вариационной теории получены полуаналитические решения волновых уравнений для полей главного луча и импульсов. Показана эволюция ширины луча главного луча и импульса.

Ключевые слова: *q-гауссиан, лазерный импульс, вариационная теория, чистая энергия, самофокусировка.*

Introduction. Ever since the beginning of human civilization, light has continuously fascinated human beings and the investigations on light-matter interactions are as old as human civilization. However, the invention of the laser by T. H. Maiman in 1960 at Hughes Research Laboratory in California, changed the entire scenario of light-matter interactions. Laser brought the same revolution to optics that the transistor brought to electronics and the cyclotron brought to nuclear physics. Laser's distinctive qualities – its ability to generate an intense, very narrow beam of light of a single wavelength – helped in revealing the true beauty of light-matter interactions. The giant leap in laser technology during the past few decades fueled by the advent of the chirped pulse amplification technique led to a renaissance in the field of light-matter interactions by giving birth to an entirely new field of research known as laser-plasma interactions. In the past few years, this new field of laser plasma interactions has gained significant interest among researchers due to its importance in laser-driven nuclear fusion [1, 2] for viable energy production without doing any harm to global climate change. In laser-driven fusion, the goal is to deposit laser energy at a particular density in the plasma in order to derive the compression and subsequent heating of the fuel pellet [3]. If the pellet is compressed sufficiently, it may undergo fusion, leading to the release of a large amount of energy. It's as if there is a tiny hunk of the sun on Earth. For the successful realization of ICF, it is highly necessary that the fuel pellet should be heated uniformly. However, due to the nonuniform irradiance (intensity ripples) over the cross sections of the laser beams, the pellet is not heated uniformly thereby deriving an instability known as

Rayleigh Taylor instability [4–6]. Whenever a not-very-dense fluid (like air) pushes on a denser fluid (like water), it is an inherently unstable situation. If the interface between the two fluids has any imperfections, any bumps or divots, then those imperfections immediately get bigger and bigger. In ICF as we compress the pellet of deuterium, it becomes denser and denser. Long before we get it hot and dense enough to fuse, it will be much denser than whatever means we are using to compress it, whether it is particles of light or a collection of hot atoms. We are using a less-dense substance to squash and contain a much denser one, and that means we will get Rayleigh Taylor instabilities. Any tiny imperfections on the interface between the plasma and the stuff that is pushing on the plasma will immediately grow. Even an almost perfectly round sphere of deuterium will quickly become distorted, squirting tendrils in all directions (Fig. 1). Just as this ruins any attempt to keep water in an inverted glass by means of air pressure, it seriously damages a machine's ability to compress and contain a plasma by means of light. Thus, it becomes essential to investigate the behavior of intensity ripples over the cross section of laser beams during their propagation through plasmas.

Intensity ripples in a laser system are due to spontaneous emissions [7, 8]. Each spontaneously emitted photon adds to the coherent field (established by stimulated emissions) a small field component whose phase is random, and thus perturbs both amplitude and phase in a random manner. The net result is that the intensity profile of the laser beam exhibit fluctuations in the form of ripples (Fig. 2).

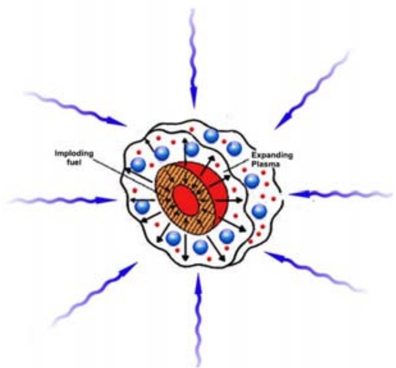

Fig. 1. Rayleigh Taylor instability in ICF.

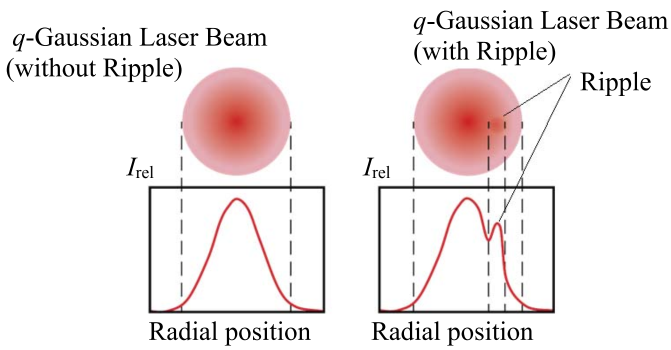

Fig. 2. Intensity ripple on laser beam.

It is a well-known fact that laser beams differing in intensity profiles behave differently in plasmas [9–12]. However, a literature review reveals the fact that most of the earlier investigations on the self-focusing of rippled laser beams in plasmas have been carried out for ideal Gaussian laser beams [13–18]. In contrast to this picture, the experimental investigations on the irradiance profile of Vulcan petawatt laser at Rutherford Appleton laboratory [19] reveal that the actual irradiance over the cross section of the laser beam is not ideally Gaussian. A significant amount of laser energy was found to be lying outside the full-width half maximum of the laser beam. By fitting into the experimental data, it has been found that the actual irradiance profile [20] of the laser beams can be modelled by Tsalli's q-Gaussian distribution [21]. To the best of the author's knowledge, earlier investigations on self-focusing of rippled laser beams in plasmas have been reported for q-Gaussian laser beams with elliptical cross sections. Thus, the aim of this paper is to give the first theoretical investigation on self-focusing of rippled q-Gaussian laser beams with elliptical cross section in plasma with an axial density ramp.

Relativistic nonlinearity of plasma. Consider the propagation of a laser beam with angular frequency ω_0 and wave number k_0 through a plasma whose equilibrium electron density is modeled as

$$n_e(z) = n_0(1+\tan(dz)), \tag{1}$$

i.e., the equilibrium density of the plasma is in the form of an upward ramp. Here, n_0 is the density of plasma electrons at $z = 0$ and the constant d is associated with rate of increase in electron density with distance. Thus, the constant d is termed as the slope of density ramp. The dielectric function of plasma modeled by Eq. (1) can be written as

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega_0^2}\left(1+\tan(dz)\right),$$

where

$$\omega_p^2 = \frac{4\pi e^2}{m_e} n_0 \tag{2}$$

is the equilibrium plasma frequency, m_e and e are the electronic mass and charge, respectively.

The laser beam has sharp intensity ripples over its cross section. The electric field vector of such a laser beam can be written as

$$\boldsymbol{E} = (E_0 + E_r)e^{\iota(k_0 z - \omega_0 t)}\mathbf{e}_x, \tag{3}$$

where E_0 is the amplitude of the field of main beam and E_r is that of the intensity ripple. Under the intense field of the laser beam, the oscillations of the plasma electrons become relativistic and the mass m_e of the electron in Eq. (2) needs to be replaced by $m_0\,\gamma$ where, m_0 is the rest mass of electron and γ is the relativistic Lorentz factor and is related to laser field amplitude as [22]:

$$\gamma = \sqrt{1+\beta EE^\star}, \tag{4}$$

where $\beta = \dfrac{e^2}{m_0^2\omega_0^2c^2}$ is the coefficient of relativistic nonlinearity and

$$EE^\star = E_0E_0^\star + E_rE_r^\star + 2E_0E_r\cos(\theta_p), \tag{5}$$

θ_p is the phase difference between the fields of the main beam and the intensity ripple.

Thus, in the presence of laser beam, the dielectric function of plasma gets modified as

$$\varepsilon = 1 - \frac{\omega_{p0}^2}{\omega_0^2}\left(1+\tan(dz)\right)\left(1+\beta A_0A_0^\star\right)^{-1/2}, \tag{6}$$

where $\omega_{p0}^2 = \dfrac{4\pi e^2}{m_0} n_0$ is the equilibrium plasma frequency. Thus, the relativistic effects make the index of refraction of plasma intensity dependent which is in turn due to the spatial dependence of the amplitude structure of the laser beam, resembling that of graded index fiber. Separating the dielectric function of plasma into linear (ε_0) and nonlinear (ϕ) parts as

$$\varepsilon = \varepsilon_0 + \phi(EE^*) \tag{7}$$

we get

$$\varepsilon_0 = 1 - \omega_{p0}^2/\omega_0^2, \tag{8}$$

$$\phi(EE^\star) = \frac{\omega_{p0}^2}{\omega_0^2}\left\{1-\left(1+\tan(dz)\right)\left(1+\beta EE^\star\right)^{-1/2}\right\}. \tag{9}$$

Evolution of beam widths of laser beam. The wave equation governing the evolution of amplitude E_0 of the main beam is

$$i\frac{\partial E_0}{\partial z} = \frac{1}{2k_0}\nabla_\perp^2 E_0 + \frac{k_0}{2\varepsilon_0}\phi\left(E_0E_0^\star\right)E_0. \tag{10}$$

Being nonlinear in nature, linear combination of two solutions is not a solution of this equation. In other words, superposition principle does not apply to eq. (10). Due to the nonapplicability of the superposition principle, eq.10 does not possess any closed form analytical solution. The only way to get physical insight is to use numerical methods or semi-analytical methods. In the present investigation, we have used a semi-analytical technique known as the variational method [23, 24] to obtain the solution of Eq. (10). This method converts the problem of solving a partial differential equation to that of solving a set of coupled ordinary differential equations. These ordinary differential equations govern the evolution of the various parameters of interest. In the case of the self-focusing of a laser beam, the parameters of interest are the beam widths of the laser beam. The essence of the method consists in finding solutions for this class of function $E_0(x,y,\sigma)$, where the set of parameters $\sigma = (f_x(z), f_y(z))$ depends on the evolution variable and is determined based on the solutions of the corresponding system of ordinary differential equations. According to this method, Eq. (10) is a variational problem for action principle based on Lagrangian density

$$\mathcal{L}_M = i\left(E_0\frac{\partial E_0^\star}{\partial z} - E_0^\star\frac{\partial E_0}{\partial z}\right) + \left|\nabla_\perp E_0\right|^2 - \frac{\omega_{p0}^2}{c^2}\int^{E_0E_0^\star}\left\{1-\left(1+\tan(dz)\right)\left(1+\beta E_0E_0^\star\right)^{-1/2}\right\}d\left(E_0E_0^\star\right). \quad (11)$$

In the present investigation we have considered the trial function of the form

$$E_0(x,y,z) = \frac{E_{00}}{\sqrt{f_xf_y}}\left\{1+\frac{1}{q}\left(\frac{x^2}{a^2f_x^2}+\frac{y^2}{b^2f_y^2}\right)\right\}^{-q/2}. \quad (12)$$

Here E_{00} is the axial amplitude of the field of the laser beam and a, b are the widths of the laser beam in x, y directions, respectively. The phenomenological parameter q is related to the deviation of the amplitude structure from the ideal Gaussian profile and is termed the deviation parameter. The value of deviation parameter q varies from laser to laser and can be obtained by fitting into the experimental data for a given laser system. f_x, f_y are the currently undetermined, real functions of only the longitudinal coordinate z. Upon multiplication with a and b, respectively, f_x and f_y give the instantaneous beam widths of the laser pulse in x and y directions respectively. Thus, f_x and f_y are termed as dimensionless beam width parameters.

Substituting the trial function given by Eq. (12) in Lagrangian density and integrating over the entire cross section of the laser beam we get the reduced Lagrangian as $L_M = \int\mathcal{L}_M d^2r$. The corresponding Euler-Lagrange equation

$$\frac{d}{dz}\left(\frac{\partial L_M}{\partial\left(\frac{\partial f_{x,y}}{\partial z}\right)}\right) - \frac{\partial L_M}{\partial f_{x,y}} = 0 \quad (13)$$

gives

$$\frac{d^2f_x}{dz^2} = \frac{1}{2k_0^2a^4}\frac{1}{f_x^3}(1-1/q)(1-2/q)\left[(1+1/q)^{-1} + \left(\langle L_1\rangle\frac{\langle L_1\rangle}{E_{00}^2}f_xf_y + 2\frac{E_{00}^2}{f_x^2f_y}\frac{\partial\langle L_1\rangle}{\partial f_x}\right)\right], \quad (14)$$

$$\frac{d^2f_y}{dz^2} = \frac{1}{2k_0^2b^4}\frac{1}{f_y^3}(1-1/q)(1-2/q)\left[(1+1/q)^{-1} + \left(\langle L_1\rangle\frac{\langle L_1\rangle}{E_{00}^2}f_xf_y + 2\frac{E_{00}^2}{f_xf_y^2}\frac{\partial\langle L_1\rangle}{\partial f_y}\right)\right], \quad (15)$$

where

$$\langle L_1\rangle = \frac{\omega_{p0}^2}{c^2}\int\left(\int_0^{E_0E_0^\star}\left\{1-\left(1+\tan(dz)\right)\left(1+\beta E_0E_0^\star\right)^{-\frac{1}{2}}\right\}d\left(E_0E_0^\star\right)\right)d^2r,$$

$$d^2r = dxdy.$$

Thus, by using Eq. (12) in Eqs. (14) and (15), we get

$$\frac{d^2f_x}{d\xi^2} = \frac{(1-1/q)(1-2/q)}{(1+1/q)}\frac{1}{f_x^3} - (1-1/q)(1-2/q)\left(\frac{\omega_{p0}^2a^2}{c^2}\right)\left(1+\tan(d'\xi)\right)\frac{\beta E_{00}^2}{f_x^2f_yg}I, \quad (16)$$

$$\frac{d^2f_x}{d\xi^2} = \left(\frac{a}{b}\right)^4\frac{(1-1/q)(1-2/q)}{(1+1/q)}\frac{1}{f_y} - \left(\frac{a}{b}\right)^2(1-1/q)(1-2/q)\left(\frac{\omega_{p0}^2a^2}{c^2}\right)\left(1+\tan(d'\xi)\right)\frac{\beta E_{00}^2}{f_x{f_y}^2}I, \quad (17)$$

where $I=\int_0^\infty u^3\left(1+\frac{u^2}{q}\right)^{-2q-1}\left(1+\frac{\beta E_{00}^2}{f_x f_y}\left(1+\frac{u^2}{q}\right)^{-q}\right)^{-\frac{3}{2}}du$, $d'=dk_0a^2$, $\xi=z/k_0a^2$.

Equations (16) and (17) are the nonlinearly coupled differential equations governing the evolution of beam widths of the elliptical q-Gaussian laser beam during its journey through the plasma. The first terms on the right-hand sides of these equations correspond to the linear propagation of the laser beam, i. e., its propagation through a vacuum or through the media whose index of refraction is independent of the intensity of the laser beam. The second terms on the right hands side (RHS) of these equations correspond to the nonlinear response of the medium. It can be seen that although in linear media the beam widths along the two transverse directions of the laser beam evolve independently, in the case of plasma due to the laser-induced optical nonlinearity, however, they get coupled to each other.

Evolution of beam width of intensity ripple. Wave equation for the intensity ripple over the cross section for the laser beam is given by

$$i\frac{\partial E_r}{\partial z}=\frac{1}{2k_0}\nabla_\perp^2 E_r+\frac{k_0}{2\varepsilon_0}\phi\left(EE^\star\right)E_r+\frac{k_0}{2\varepsilon_0}\left[\phi\left(EE^\star\right)-\phi\left(E_0E_0^\star\right)\right]E_0. \quad (18)$$

The Lagrangian density corresponding to this equation can be written as

$$\mathcal{L}_R=i\left(E_r\frac{\partial E_r^\star}{\partial z}-E_r^\star\frac{\partial E_r}{\partial z}\right)+\left|\nabla_\perp E_r\right|^2-\frac{\omega_{p0}^2}{c^2}\int^{E_rE_r^\star}\left\{1-\left(1+\tan\left(dz\right)\right)\left(1+\beta E_rE_r^\star\right)^{-1/2}\right\}d\left(E_rE_r^\star\right)-$$
$$-\frac{\omega_{p0}^2}{c^2}\int^{E_0E_0^\star}\left(\left\{1-\left(1+\tan(dz)\right)\left(1+\beta E_rE_r^\star\right)^{-\frac{1}{2}}\right\}-\left\{1-\left(1+\tan(dz)\right)\left(1+\beta E_0E_0^\star\right)^{-1/2}\right\}\right)d\left(E_0E_0^\star\right). \quad (19)$$

In the present investigation we have assumed Gaussian irradiance profile of the intensity ripple riding over the cross section of the laser beam. Such an intensity ripple can be modeled as

$$E_\gamma E_\gamma^*=\frac{E_{r00}^2}{g^2}e^{-\left(\frac{x^2+y^2}{r_\gamma^2g^2}\right)}\left(\frac{x^2+y^2}{r_\gamma^2g^2}\right)^n. \quad (20)$$

Here r_r is the initial width of the ripple and g is the dimensionless beam width parameter of the ripple. The constant n gives the position of intensity ripple from the axis of the main beam. As the value of n increases, the intensity ripple shifts away from the beam axis.

Using the same procedure as that of below 3, we get the following equation for the evolution of beam width of the intensity ripple

$$\frac{d^2g}{d\xi^2}=\left(\frac{a}{r_\gamma}\right)^4\frac{1}{(n+1)g^3}-\left(\frac{\omega_{p0}}{c}\right)^2\left(\frac{a}{r_\gamma}\right)^4\frac{1}{g\Gamma(n+1)}\left(2\beta E_{00}^0\left(\left(\frac{a}{r_\gamma}\right)^{2n}\left(\frac{f_x}{g}\right)^{2n+2}+\left(\frac{b}{r_\gamma}\right)^{2n}\left(\frac{f_y}{g}\right)^{2n+2}\right)\right)R_1+$$
$$+\beta E_{00}E_{r00}\left(\left(\frac{a}{r_\gamma}\right)^n\left(\frac{f_x}{g}\right)^{n+1}+\left(\frac{b}{r_\gamma}\right)^n\left(\frac{f_y}{g}\right)^{n+1}\right)\cos\theta_pR_2-\beta E_{00}E_{r00}\left(\left(\frac{a}{r_\gamma}\right)^n\left(\frac{f_x}{g}\right)^{n+1}+\left(\frac{b}{r_\gamma}\right)^{2n}\left(\frac{f_y}{g}\right)^{2n+2}\right)\cos\theta_pR, \quad (21)$$

where

$$R_1=\int_0^{2\pi}\int_0^\infty u^{2n+1}G\left(E_{00},E_{r00}\right)e^{-\frac{1}{2}\left(\frac{a^2f_x^2}{r_r^2g^2}\cos^2\theta+\frac{b^2f_y^2}{r_r^2g^2}\sin^2\theta\right)u^2}\left[n+1-\frac{a^2f_x^2}{r_r^2g^2}\cos^2\theta+\frac{b^2f_y^2}{r_r^2g^2}\sin^2\theta u^2\right]dud\theta,$$

$$R_2=\int_0^{2\pi}\int_0^\infty u^{n+1}G\left(E_{00},E_{r00}\right)e^{-\frac{1}{2}\left(\frac{a^2f_x^2}{r_r^2g^2}\cos^2\theta+\frac{b^2f_y^2}{r_r^2g^2}\sin^2\theta\right)u^2}\left(1+\frac{u^2}{q}\right)^{-\frac{q}{2}}\left[n+2-\left(1+\frac{a^2f_x^2}{r_r^2g^2}\cos^2\theta+\frac{b^2f_y^2}{r_r^2g^2}\sin^2\theta\right)u^2\right]dud\theta,$$

$$R_3=\int_0^{2\pi}\int_0^{\infty}u^{2n+1}e^{-1/2\left(\frac{a^2f_x^2}{r_r^2g^2}\cos^2\theta+\frac{b^2f_y^2}{r_r^2g^2}\sin^2\theta\right)u^2}\left(1+u^2/q\right)^{-q/2}\left[1-\frac{1}{\left(1+\frac{\beta E_{00}^2}{f_xf_y}(1+u^2/q)\right)^{1/2}}(1+\tan(d'\xi))\right]\times$$

$$\times\left[n+2-\frac{1}{2}\left(1+\frac{a^2f_x^2}{r_r^2g^2}\cos^2\theta+\frac{b^2f_y^2}{r_r^2g^2}\sin^2\theta\right)u^2\right]dud\theta,$$

$$G\left(E_{00},E_{r00}\right)=\left[1-\frac{1}{\left(1+X(f,g)\right)^{\frac{1}{2}}}\left(1+\tan(d'\xi\right)\right],$$

$$X(f_x,f_y,f_z)=\frac{\beta E_{00}^2}{f_xf_y}(1+u^2/q)^{-q}+\frac{\beta E_{r00}^2}{g^2}\left(\frac{a^2f_x^2}{r_r^2g^2}\cos^2\theta+\frac{b^2f_y^2}{r_r^2g^2}\sin^2\theta\right)^n u^{2n}e^{-\left(\frac{a^2f_x^2}{r_r^2g^2}\cos^2\theta+\frac{b^2f_y^2}{r_r^2g^2}\sin^2\theta\right)u^2}+$$

$$+2\frac{\beta E_{00}E_{r00}}{\sqrt{f_xf_yg}}\cos\theta_p\left(\frac{a^2f_x^2}{r_r^2g^2}\cos^2\theta+\frac{b^2f_y^2}{r_r^2g^2}\sin^2\theta\right)^{n/2}u^ne^{-\frac{1}{2}\left(\frac{a^2f_x^2}{r_r^2g^2}\cos^2\theta+\frac{b^2f_y^2}{r_r^2g^2}\sin^2\theta\right)u^2}(1+u^2/q)^{-q}{}^{/2}.$$

Results and discussion. In the present investigation the Runge Kutta fourth-order method has been used to solve Eqs. (16), (17), and (21) numerically for the following set of laser-plasma parameters:

$\omega_0=1.78\times10^{15}$ rad/s, $a=10\mu$m, $\beta E_{00}^2=3$, $\frac{\omega_{p0}^2a^2}{c^2}=9$, $\beta E_{r00}^2=0.25$, $\frac{r_r}{a}=0.1$, $\theta_p=\frac{\pi}{6}$, $n=1$ and q = (3, 4, ∞), d'=(0.025, 0.035, 0.045) and a/b = (1, 1.1, 1.2).

In solving Eqs. (16)–(18), it has been assumed that at the plane of incidence, the laser beam has a plane wave-front. Mathematically this condition means that at ξ = 0:

$$f_{x,y}=g=1,$$

$$\frac{df_{x,y}}{d\xi}=\frac{dg}{d\xi}=0.$$

Figure 3 illustrates the evolution of beam widths of the laser beam and that of the intensity ripple with the distance of propagation through the plasma. It can be seen that inside the plasma medium, the beam widths of the main beam along both of the transverse directions show oscillatory behavior across the longitudinal direction. The oscillatory variations of the beam widths of the laser beam are due to the saturation nature of the relativistic nonlinearity of plasma. Initially, due to laser-induced relativistic nonlinearity, the beam widths of the laser beam start decreasing and hence its intensity starts increasing, which further enhances the relativistic nonlinearity. When the intensity of the laser beam becomes too high, the mass of plasma electrons in the illuminated portion of the plasma becomes saturated. Hence, now the laser beam propagates as if it is propagating through a vacuum; thus, after attaining the minimum possible beam widths, the spot size of the laser beam bounces back. These processes keep on repeating themselves giving an oscillatory behavior to the beam widths of the laser beam.

Further it has been observed that, after every focal spot, the maximum as well as the minimum of the beam width shift downwards. This is owing to the fact that the equilibrium electron density is an increasing function of longitudinal distance. Hence, the plasma index of refraction keeps on decreasing with the penetration of the laser beam into the plasma. Consequently, the self-focusing effect gets enhanced and the maximum as well as minimum of the beam width go on shifting downwards after every focal spot. It is also seen that the frequency of the oscillations of the beam width increases with distance. The physics behind this fact is that the denser the plasma, the higher the phase velocity of the laser beam through it. Hence, in denser plasma, the laser beam takes less time to become self-focused.

It can be also be seen that initially the beam widths of the laser beam along the two transverse directions vary in phase with each other but over some distance of propagation their oscillations establish a phase mismatch. This phase mismatch in the oscillations of beam widths along the x and y directions is due to the fact that, because of its ellipticity, the laser beam experiences different indices of refraction along the x and y directions. i. e., for the elliptical beam the plasma behaves as an anisotropic medium.

It can also be seen that the extent of self-focusing of the laser beam along the x direction is more compared to that along the y direction, which is due to the fact that the initial width of the beam along the x direction is more compared to that along the y direction ($a/b = 1$, $a/b = 1.1$). Thus, the opposition offered by the diffraction effect to the nonlinear refraction is more along the y direction, resulting in reduced focusing along the y direction.

The Fig. 3 indicates that the beam width of the intensity ripple decreases monotonically with the distance of propagation, showing step-like behavior at the focal spots of the laser beam. This is due to the fact that because of its nonlinear coupling with the main beam, the intensity ripple follows the main beam. Thus, as the main beam gets self-focused, the beam width of the intensity ripple also starts decreasing. However, after attaining minimum beam width and the spot size of the main beam bounces back, the ripple does not follow it but rather its beam width keeps on decreasing with the distance of propagation. This is due to the fact that the intensity of the laser ripple is not enough to saturate the relativistic nonlinearity. The step-like behavior of the beam width of intensity ripple at the focal spots of the main beam is due to the fact that at these locations the intensity ripple experiences maximum relativistic nonlinearity. Thus, at the focal spots of the main beam, the rate of change of beam width of the ripple experiences a sudden increase, leading to step-like behavior in the beam width of the ripple.

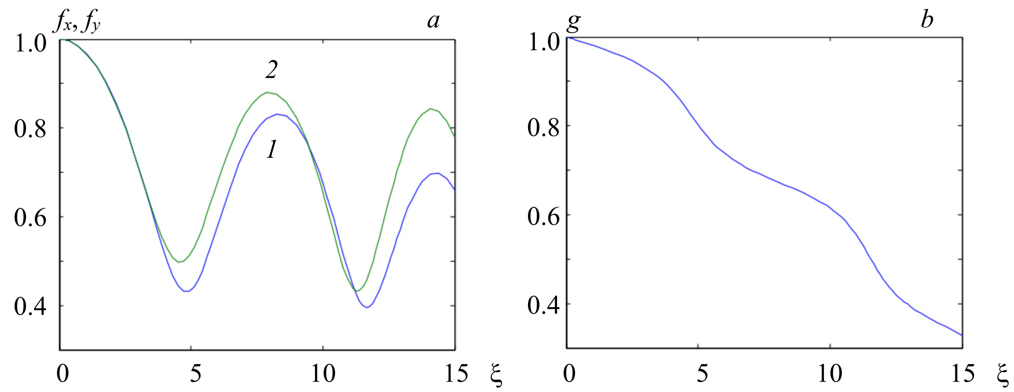

Fig. 3. (a) Evolution of beam widths f_x (1), f_y (2) of the laser beam and (b) g of the intensity ripple with distance of propagation through plasma for $q = 3$, $d' = 0.025$, and $a/b = 1.1$.

Now in order to see the effect of the deviation parameter q on the evolution of beam widths of the laser beam and that of intensity ripple, Eqs. (16), (17), and (21) have been solved for different values of q, while keeping other laser-plasma parameters fixed. The corresponding variations of the beam widths of the main beam and that of the ripple are shown in Fig. 4. It can be seen that with increase in the value of deviation parameter q, the extent of self-focusing along both the transverse directions is reduced. This is due to the fact that for laser beams with a larger value of q, most of the beam energy is concentrated around a narrow region around the beam axis. Hence, these beams get a little contribution form the of axial rays towards the nonlinear refraction. As the phenomenon of self-focusing is a homeostasis of nonlinear refraction of the optical beam due to optical nonlinearity of the medium, increase in the value of deviation parameter q reduces the extent of self-focusing of the laser beam. Thus, compared to q-Gaussian laser beams, ideal Gaussian laser beams possess minimum focusing character.

It can also be seen that instead of their reduced focusing, laser beams with higher values of deviation parameter q possess faster focusing along both of the transverse directions, due to the faster focusing character of the rays closer to beam axis. Being away from the beam axis, axial rays take longer to get self-focused. As there are more off axial rays in laser beams with lower values of deviation parameter q, these beams possess slower focusing character.

It can also be seen that increase in the deviation parameter q of the main beam results in decrease in the extent of self-focusing of the intensity ripple. This is due to the nonlinear coupling of the intensity ripple with the main beam. As a result of this nonlinear coupling between the main beam and ripple, there is one to one correspondence between the self-focusing of the main laser beam and that of the ripple. Thus, as increase in the value of q results in a decrease in the extent of self-focusing of the main beam, there is a further decrease in the extent of the self-focusing of laser ripple with the increase in the value of the deviation parameter q.

Figure 5 illustrates the effect of beam ellipticity on self-focusing of main beam as well as that of the intensity ripple. It can be seen that with the increase in the beam ellipticity along the y direction, there is a reduction in the extent of self-focusing of the laser beam along the y direction. This is due to the fact that, at a fixed value of a, increase in beam ellipticity (i. e., a/b) means the reduction in initial width of the beam along the y direction. Hence, the increase in beam ellipticity along the y direction makes the diffraction effect stronger along the y direction. This results in the reduced focusing of the laser beam along the y direction.

It can also be seen that initially the increase in beam ellipticity does not produce any significant effect on self-focusing of the beam along the x direction. However, as the beam penetrates deeper into the plasma, the focusing along the x direction also decreases, due to the fact that as the beam penetrates deeper and deeper into the plasma, the nonlinear coupling between the two beam widths becomes stronger and stronger.

From the Fig. 5 it can also be seen that with increase in ellipticity of the main beam, there is decrease in the rate of decrease of beam width of the ripple. This is due to the fact that with increase in ellipticity of the main beam, the overall extent of it self-focusing gets reduced. As the extent of self-focusing of ripple is dependent on that of the main beam, decrease in the extent of self-focusing of the main beam results in a decrease in that of the ripple.

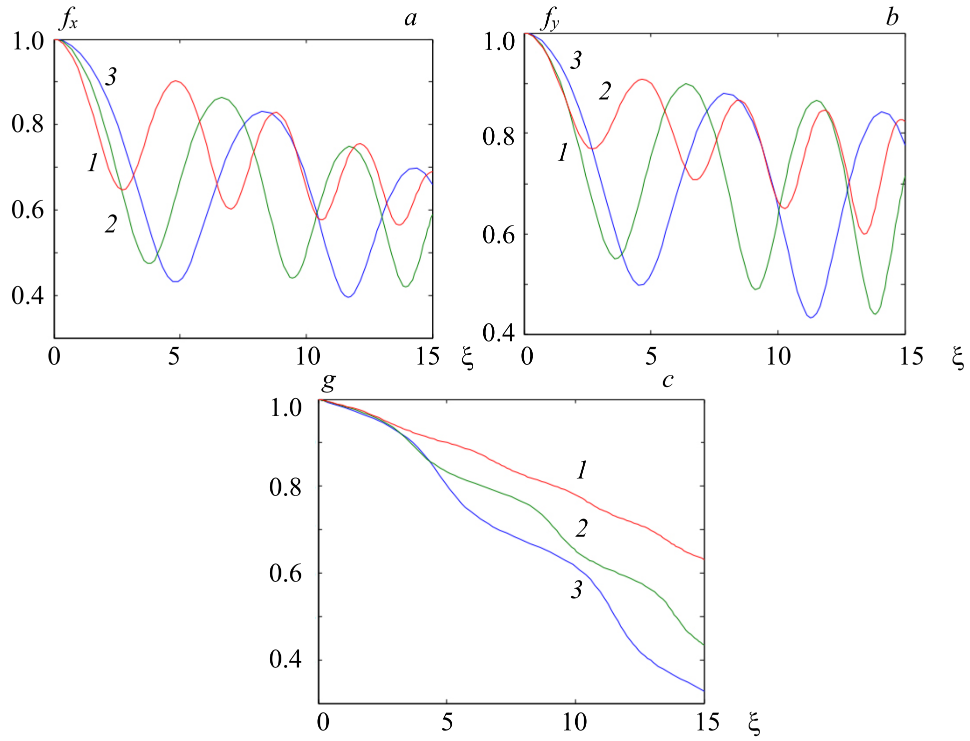

Fig. 4. Evolution of beam widths (a) f_x, (b) f_y of the laser beam and (c) g of the intensity ripple with distance of propagation through plasma for $q = 3$ (1), 4 (2), ∞ (3), $d' = 0.025$ and $a/b = 1.1$.

Figure 6 depicts the effect of the slope of the density ramp on the extent of the self-focusing of the main beam as well as that of the intensity ripple riding over its cross section. It can be seen that increase in slope of the density ramp enhances the extent of self-focusing of the main beam along both of the transverse directions. This is due to the fact that with increase in slope density of the ramp, the number of electrons contributing to the relativistic nonlinearity increases along the direction of propagation. This results in enhanced transverse as well as longitudinal gradient in the index of refraction of the plasma that in turn increases the extent of self-focusing of the laser beam along the two transverse directions.

The plots in Fig. 6 also indicate an increase in the extent of the self-focusing of the ripple with an increase in the slope of the density ramp. This is also due to increase in the extent of self-focusing of the main beam with an increase in the slope of the density ramp.

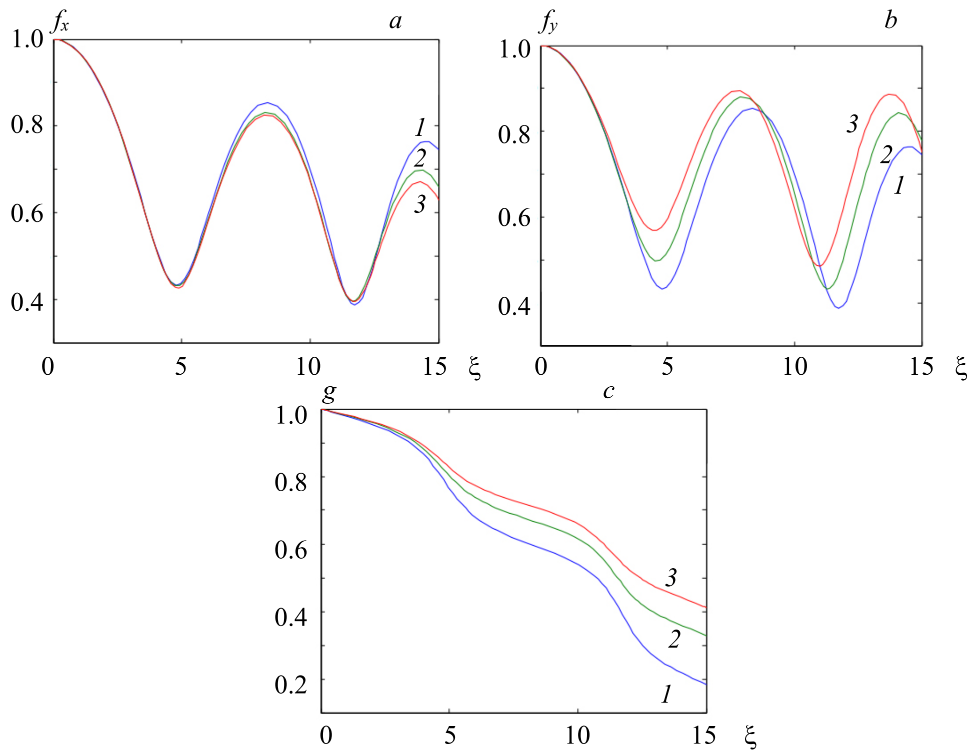

Fig. 5. Evolution of beam widths (a) f_x, (b) f_y of the laser beam and (c) g of the intensity ripple with distance of propagation through plasma for $q = 3$, $d' = 0.025$ and $a/b = 0$ (1), 1.1 (2), 1.2 (3).

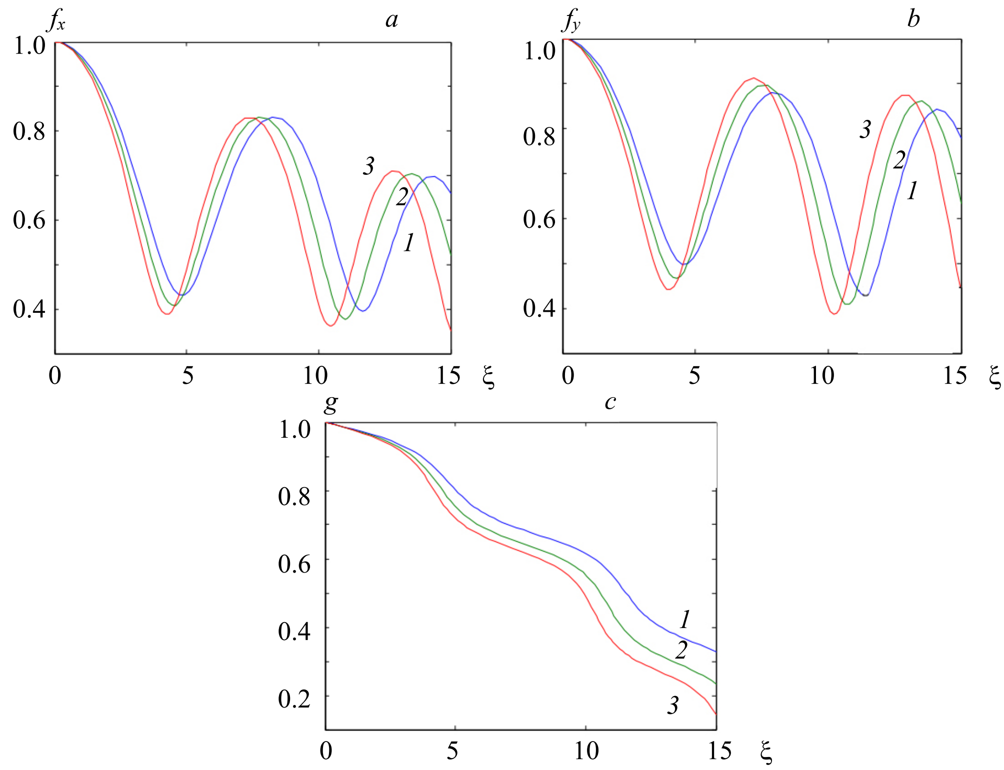

Fig. 6. Evolution of beam widths (a) f_x, (b) f_y of the laser beam and (c) g of the intensity ripple with distance of propagation through plasma for $q = 3$, $d' = 0.025$ (1), 0.035 (2), 0.045 (3) and $a/b = 1.1$.

Conclusions. Effect of self-focusing of the laser beam on propagation dynamics of intensity ripples riding over its cross section has been investigated. It can be concluded that as the irradiance profile of the main laser beam converges towards the ideal Gaussian profile, the rate of localization of the intensity ripple reduced. Thus, in order to obviate the risk of Rayleigh Taylor instability in ICF, the irradiance over the cross sections of the laser beams should be close to the ideal Gaussian profile. Another way to negate the Rayleigh Taylor instability is to make the laser beams slightly elliptical, as with an increase in beam ellipticity, the rate of the localization of the intensity ripples reduces.

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