ITERATIVE FREQUENCY-DOMAIN INTERFEROMETRY TECHNOLOGY FOR ULTRAFAST PHASE MEASUREMENT

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This paper presents an original design of the single-shot iterative frequency-domain interferometry (IFDI) technology to measure the ultrafast phase. Unlike frequency domain holography (FDH), in which the reference pulse interferes with the phase modulated probe pulse, in IFDI two linearly chirped probe pulses co-propagate and are both phase-modulated by the measured ultrafast phase, and then the phase can be reconstructed with the iterative algorithm. Compared with two types of FDH, the IFDI technology has better accuracy and stability.

Keywords: *iterative frequency-domain interferometry, single shot measurement, time-resolved imaging.*

ИТЕРАЦИОННАЯ ЧАСТОТНАЯ ИНТЕРФЕРОМЕТРИЯ ДЛЯ ИЗМЕРЕНИЯ СВЕРХБЫСТРОЙ ФАЗЫ

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Представлена технология однократной итеративной интерферометрии в частотной области (IFDI) для измерения сверхбыстрой фазы. В отличие от голографии в частотной области (FDH), в которой опорный импульс интерферирует с объектным фазомодулированным импульсом, в IFDI два линейно чирпированных объектных импульса распространяются совместно и оба модулированы по фазе со сверхбольшой частотой, причем эта фаза может быть восстановлена с помощью итеративного алгоритма. По сравнению с двумя типами FDH технология IFDI имеет лучшие точность и стабильность.

Ключевые слова: *итерационная интерферометрия в частотной области, измерение единичного кадра, визуализация с временным разрешением.*

Introduction. Direct observation of the dynamic interplay among drive pulse, plasma wave, and accelerated electrons, including wave-breaking, pump depletion, and beam loading, is essential for realizing potential applications of plasma accelerators in many fields such as radiobiology, radiotherapy, radiographic materials inspection, ultrafast chemistry, and high energy physics. As a single-shot interferometric technique, frequency domain holography (FDH) can measure the ultrafast phase caused by laser-generated nonlinear refractive index structures. FDH opens a direct window into microscopic physics and is an essential step towards controlling the above-mentioned dynamics.

Using FDH, Le Blanc et al. realized a single-shot time-resolved measurement of the ultrafast phase shifts induced either by the nonlinear susceptibility χ^3 of fused silica or by ionization fronts in air over a temporal region of 1 ps with a 70 fs resolution [1]. Then, using FDH, Matlis et al. demonstrated a single-shot visualization of laser-wake field accelerator structures for the first time [2]. Li et al. experimental results illustrate both the strengths (fast, faithful single-shot imaging of most aspects of the wake structure) and limitations (underestimate of plasma oscillation amplitude when the plasma structure evolves significantly, false structure from pump-generated radiation) of FDH imaging [3]. Furthermore, Li et al. presented a generalization of FDH known as frequency domain tomography (FDT), which incorporates several FDHs having different angle between probe pulse and drive pulse, to visualize evolving light-velocity objects [4].

However, there are some application limits of FDH. In this paper, FDH was divided into two types, and the limits of FDH were analyzed. Here, a new design of the iterative frequency-domain interferometry (IFDI) technology was proposed, and a comparison between FDH and IFDI confirmed the feasibility and accuracy of IFDI technology.

FDH limitation analysis and IFDI technology. In FDH, a wide-bandwidth, temporally extended probe pulse co-propagates with the drive pulse through the medium, illuminating the entire object being measured at once, and its phase is modulated by the object. The temporally advanced reference pulse, which does not overlap with the object, has the same time-frequency characteristic as the linearly chirped probe pulse. The reference pulse interferes with the phase modulated probe pulse at the detection plane of an imaging spectrometer.

Figure 1a shows the schematic illustration of FDH; here τ is the time delay between reference pulse and probe pulse, and the spectrum interference fringes can be expressed as

$$I(\omega) = |E_{\text{ref}}(\omega)|^2 + |E_{\text{probe}}(\omega)|^2 + 2|E_{\text{ref}}(\omega)||E_{\text{probe}}(\omega)|\cos[\Delta\varphi(\omega) - \omega\tau], \tag{1}$$

where $|E_{\text{probe}}(\omega)|$ and $|E_{\text{ref}}(\omega)|$ are the spectral amplitudes of the probe pulse and the reference pulse, respectively; $\Delta\varphi(\omega)$ is the spectral phase difference between the probe pulse $\varphi_{\text{probe}}(\omega)$ and the reference pulse $\varphi_{\text{ref}}(\omega)$, which can be reconstructed from the spectrum interference fringes. Let us assume that the variation of the temporal phase modulation is sufficiently slow compared with the sampling interval, and the time-domain phase shifts of the linearly chirped probe pulse correspond to the frequency-domain phase shifts of different instantaneous frequencies; then the phase shifts $\Delta\varphi(t)$ can be reconstructed from $\Delta\varphi(\omega)$ by Fourier transform methods.

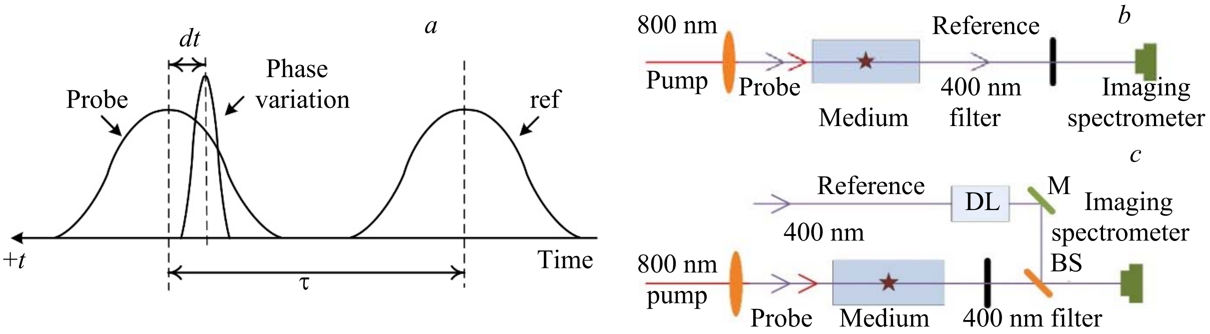

Fig. 1. Schematic illustration of FDH (a), and arrangements of type I FDH (b) and type II FDH (c).

According to whether or not the reference pulse propagates through the medium, the FDH can be divided into type I (reference and probe pulses co-propagate through the medium) and type II (reference pulse does not propagate through the medium as the probe pulse). The scheme of Matlis et al., as shown in Fig. 1b, in which two linearly chirped, frequency-doubled pulses, reference and probe pulses, co-propagate through the medium with the drive (or pump) pulse, is a typical type I FDH, and the reconstructed phase $\Delta\varphi(\omega)$ is just the phase $\Delta\Phi(\omega)$ to be measured.

The spectral interference fringe width can be expressed as $\Delta\omega_{\text{fringes}} = 2\pi/\tau$, where τ is the time delay between the reference and probe pulses. In the type I FDH, the reference and probe pulses co-propagate through the medium, so τ must be larger than the time duration of the ultrafast phase T_p or the time $T = L/v$, where L is the length of the object along the direction of propagation and v is the propagation velocity of the probe pulse. Meanwhile, $\Delta\omega_{\text{fringes}}$ can also be expressed in terms of the spectral resolution of the imaging spectrometer $\Delta\omega_{\text{res}}$ by $\Delta\omega_{\text{fringes}} = N\Delta\omega_{\text{res}}$. According to the Nyquist-Shannon a ampling theorem, N should be ≥ 2, and for complicated phase perturbation, N should be larger. Obviously, when T_p or T is too large, $\Delta\omega_{\text{fringes}}$ will be too small to be completely resolved by the spectrometer and will cause further reconstructed phase error. For example, assume that the wavelength λ is 400 nm and the resolution of the imaging spectrometer is 0.05 nm;

then the spectral resolution $\Delta\omega_{res} = 2\pi \times 0.09375$ THz. Assume that $T_p = 10$ ps, approximately the time duration of the terahertz waveforms [5]; then the maximum width of the fringes is $\Delta\omega_{fringes\text{-}max} = 2\pi\times0.1$ THz when τ takes the minimum value $\tau_{min} = T_p = 10$ ps in the type I FDH. So, the spectral interferogram cannot be completely resolved, and the phase perturbation cannot be reconstructed accurately.

In a type II FDH, as shown in Fig. 1c, the reference pulse does not propagate through the medium, so the time delay τ between reference and probe pulses is not limited to being larger than the time duration of the ultrafast phase T_p.

However, the reconstructed phase $\Delta\varphi(t)$ is the sum of the ultrafast phase $\Delta\Phi(t)$ to be measured and the phase $\Delta\varphi'(t)$ caused by the material dispersion of the medium, which can be expressed as $\Delta\varphi'(t) = nkz$, where n is the refractive index of the medium, z is the length of the medium along the direction of the probe propagation, and k is the wave vector. So $\Delta\Phi(t)$ reads

$$\Delta\Phi(t) = \Delta\varphi(t) - \Delta\varphi'(t), \tag{2}$$

where $\Delta\varphi'(t)$ is reconstructed under the conditions that the drive (or pump) pulse is blocked. So, the type II FDH is not limited by the time duration T_p, while it needs an extra step to measure $\Delta\varphi'(t)$. The accuracy and stability of the phase measurement is easily and greatly influenced by the environment, especially for a spatially distributed phase measurement.

Here, we present a design of the IFDI technology; Fig. 2 shows the basic principle. Twin linearly-chirped Gaussian pulses probe 1 and probe 2 co-propagate and are used to measure the ultrafast phase $\Delta\Phi(t)$, and τ is the time delay between the iterative pulses.

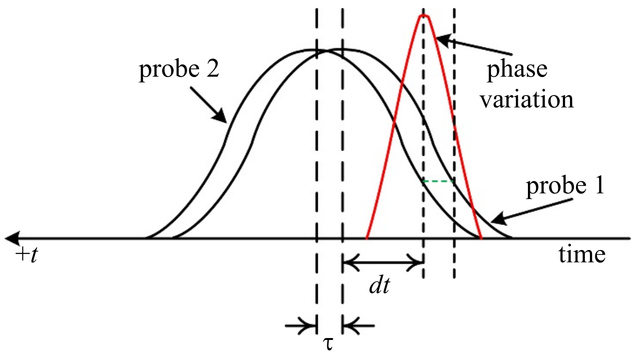

Fig. 2. Schematic illustration of IFDI.

Assume that the variation of the temporal phase modulation is sufficiently slow, and the time-domain phase shifts of the linearly chirped probe pulse correspond to the frequency-domain phase shifts of different instantaneous frequencies as in the FDH. Take ω_c, the instantaneous angular frequency component of the chirped pulse, as an example; the phase perturbation $\Delta\Phi$ gives the phase $\Delta\Phi(t_1)$ to ω_c of probe 1 at t_1 and gives the phase $\Delta\Phi(t_2)$ to ω_c of probe 2 at t_2, so the time delay $\tau = t_2 - t_1$, and the phase shifts of the ω_c of probe 1 and probe 2 can be expressed as

$$\varphi_{probe1}(\omega_c) = \Delta\Phi(\omega_c), \tag{3}$$

$$\varphi_{probe2}(\omega_c) = \Delta\Phi(\omega_c - \Omega), \tag{4}$$

where $\Omega = \tau/\varphi''$, and φ'' is the value of the quadratic dispersion of the chirped probe pulse at the center frequency ω_0. The spectral interference fringes can be expressed as

$$I(\omega) = |E_1(\omega_c)|^2 + |E_2(\omega_c)|^2 + 2|E_1(\omega_c)||E_2(\omega_c)|\cos\left[\Delta\Phi(\omega_c) - \Delta\Phi(\omega_c\text{-}\Omega) + \omega\tau\right]. \tag{5}$$

To simplify notation, define the spectral phase difference as $\theta(\omega_c) = \Delta\Phi(\omega_c) - \Delta\Phi(\omega_c - \Omega)$. To isolate $\theta(\omega_c)$, we use a robust algorithm introduced by Takeda et al. [6] in which the data is Fourier transformed with respect to the spectrometer frequency, filtered, and inverse-transformed. So $\theta(\omega_c)$ can be reconstructed just like the $\Delta\varphi(\omega)$ in the FDH.

Then the spectral phase $\Delta\Phi(\omega)$ can be reconstructed by concatenating the spectral phase difference $\theta(\omega)$. The spectral phase at some frequency, say ω_0, is set equal to zero so that $\Delta\Phi(\omega_0 - \Omega) = -\theta(\omega_0)$, and the spectral phase $\Delta\Phi(\omega)$ for all frequencies that are multiples of the Ω away from ω_0 follow in this fashion [7]:

$$
\begin{aligned}
&\vdots\\
\Delta\Phi(\omega_0-2\Omega) &= -\theta(\omega_0-\Omega)-\theta(\omega_0),\\
\Delta\Phi(\omega_0-\Omega) &= -\theta(\omega_0),\\
\Delta\Phi(\omega_0) &= 0,\\
\Delta\Phi(\omega_0+\Omega) &= \theta(\omega_0+\Omega),\\
\Delta\Phi(\omega_0+2\Omega) &= \theta(\omega_0+2\Omega)+\theta(\omega_0+\Omega).\\
&\vdots
\end{aligned} \tag{6}
$$

By simply adding up the phase differences, we can reconstruct the phase for frequencies separated by Ω. If Ω is small relative to the structure of the spectral phase ΔΦ(ω), the phase difference θ(ω) is approximately the first derivative of the spectral phase

$$\theta(\omega_c)=\Delta\Phi(\omega_c)-\Delta\Phi\left(\omega_c-\Omega\right)\approx\Omega\frac{d\Delta\Phi(\omega_c)}{d\omega_c}. \tag{7}$$

Accordingly, $\Delta\Phi(\omega_c)$ can be reconstructed by integration [6].

Simulation results and discussion. Assume that the twin linearly-chirped Gaussian pulses with the spectral bandwidth (full width at half maximum, FWHM) Δω act as the probe pulse and the time advanced reference pulse in the FDH and two-probe pulses in the IFDH. The linearly-chirped Gaussian pulse can be expressed as

$$E(t)=E_0\,\mathrm{e}^{-at^2+i\omega_0 t}\,\mathrm{e}^{ibt^2}=E_0\,\mathrm{e}^{-at^2}\,\mathrm{e}^{i(\omega_0+bt)t}, \tag{8}$$

where $a = 2\ln 2 \times \tau - 2$ chirp, τ chirp = $\Delta\omega/2b$ is the FWHM of the chirped pulse, ω_0 is the central frequency, and b is the chirp parameter which can be expressed as

$$b=\frac{1}{4}\beta_2^{-1}\left[1+(2\ln 2)^2\beta_2^{-2}(\Delta\omega)^{-4}\right]^{-1}\approx\frac{1}{4}\beta_2^{-1}\left[1+2\beta_2^{-2}(\Delta\omega)^{-4}\right]^{-1}, \tag{9}$$

where $\beta_2 = 1/2(\partial^2\varphi(\omega)/\partial\omega^2)$ is the group delay dispersion (GDD), and φ(ω) is the spectral phase.

Assume that the phase perturbation $\Delta\Phi(t)$ is to be measured, whose Taylor series expansion is $\Delta\Phi(t) = \varphi_0 + \varphi_1 t + \varphi_2 t^2$. Here we ignored third- and higher-order terms and set $\varphi_0 = \varphi_1 = 0$ to consider only the quadratic term and to simplify the simulation. The other simulation conditions are as follows: $\omega_0 = 2\pi c/\lambda_0|_{\lambda=400\mathrm{nm}} = 4.71$ rad/fs, an SF57 is used for pulse stretching with $d^2n/d\lambda^2|_{\lambda=400\mathrm{nm}} = 8.8556$ μm^{-2}, $\mathrm{GVD}|_{\lambda=400\mathrm{nm}} = 1003.6$ fs^2/mm, $\tau_{\mathrm{chirp}} = 12$ ps, $a = 9.627\times10^{-9}$ fs^{-2}, $\Delta\omega = 2\pi\times13$ THz, and $b = 1.118\times10^{-6}$ rad/fs^2. Set $\varphi_2 = a$; then the phase perturbation is $\Delta\Phi(t) = at^2$.

In the type I FDH, the profiles of the reconstructed phase and the phase perturbation $\Delta\Phi(t) = at^2$ are shown in Fig. 3a at τ = 10 ps and in Fig. 3b at τ = 1 ps. Apparently, the type I FDH cannot give an accurate ultrafast time-varying phase with relatively large T_p because of the spectrometer's resolution limits.

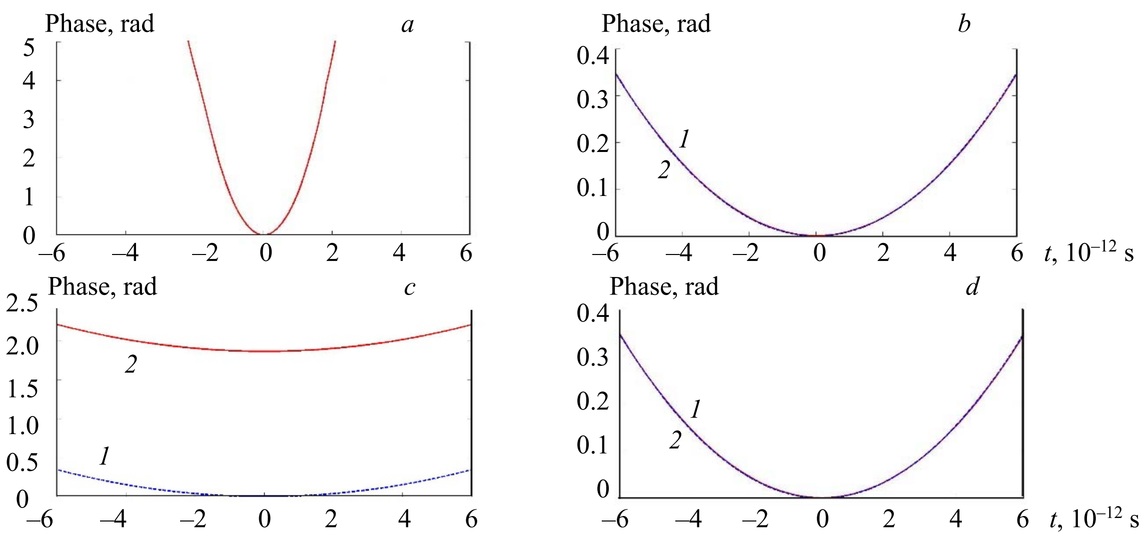

Fig. 3. The reconstructed phase (a) T_p = 10 ps and (b) T_p = 1 ps in type I and (c) Δφ(t) and (d) ΔΦ(t) in type II FDH, the original phase (1) (dashed line) and the reconstructed phase (2) (solid line).

In the type II FDH, using the same simulation conditions as in the type I FDH , set $\tau = 1.5$ ps. Assume that the medium is glass BK7, and the length of BK7 is $z = 1$ cm; then the refractive index n_{BK7} is

$$n^2_{\mathrm{BK7(SCHOTT)}} = \frac{1.03961212\lambda^2}{\lambda^2 - 0.00600069867} + \frac{0.231792344\lambda^2}{\lambda^2 - 0.0200179144} + \frac{1.01046945\lambda^2}{\lambda^2 - 103.560653} + 1. \quad (10)$$

The profiles of the reconstructed phase $\Delta\varphi(t)$ and $\Delta\Phi(t)$ and the phase perturbation are shown in Figs. 3c,d. The reconstructed phase is accurate in the type II FDH.

In the IFDI, the twin linearly-chirped Gaussian pulses, similarly to the probe pulse in the type I FDH and type II FDH, act as probe 1 and probe 2 delayed by τ, respectively. Using the same simulation conditions as in the FDH, set $\tau = 1.5$ ps and $\Delta\Phi(t) = at^2$; the simulated spectral interferogram of the interference of probe 1 and probe 2 is shown in Fig. 4a.

Then $\Delta\Phi(t)$ is reconstructed from Fig. 6, as shown in Fig. 4b. It is worth noting that the effective detection time range of the IFDI is the overlap between probe pulses ($\tau_{\mathrm{chirp}} - 2\tau$). Thus, we can see that the profile of the reconstructed phase in the middle part (effective detection time range) coincides with that of the phase perturbation $\Delta\Phi(t) = at^2$, and both edges are somewhat inaccurate compared to the type II FDH $\Delta\Phi(t)$ in Fig. 3d.

Although the effect of noise in actual experiments can be a major factor in application, because the noise effect is consistent for FDH and IFDI, it is ignored in the comparative simulation.

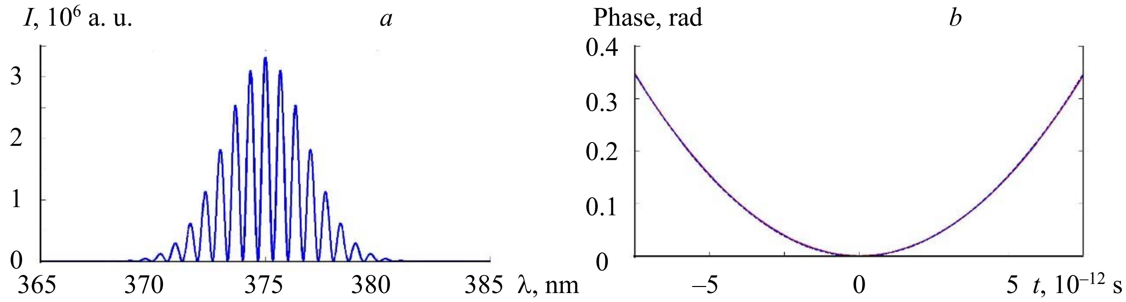

Fig. 4. The original temporal phase perturbation (solid line) and the reconstructed phase (dashed line) in IFDI.

Conclusions. As mentioned above, the type I IFDH cannot measure accurately the ultrafast phase with relatively large T_p due to the spectrometer's resolution limits. By contrast, IFDI and type II FDH can overcome the limitation and reconstruct the phase more accurately. Compared with the type II FDH, IFDI can eliminate the extra step of measuring the phase $\Delta\varphi'(t)$ caused by the material dispersion of the medium and has greater accuracy and stability, especially for spatially distributed phase measurement. However, IFDI is only appropriate for pure phase measurement.

In summary, IFDI is a simple and reliable interferometric technique for measuring the ultrafast phase variations and can realize single-shot visualization of evolving light-velocity objects, and the reconstructed result is extremely accurate. Based on the IFDI technique, we can develop new technology like FDT [4] to visualize the spatiotemporal dynamics of light-velocity objects and image a wide range of nonlinear propagation phenomena, including filament formation in gases and the evolution of plasma wake fields.

REFERENCES

1. S. P. Le Blanc, E. W. Gaul, N. H. Matlis, A. Rundquist, M. C. Downer, *Opt. Lett.*, **25**, 764–766 (2000).
2. N. H. Matlis, S. Reed, S. S. Bulanov, V. Chvykov, G. Kalintchenko, T. Matsuoka, P. Rousseau, V. Yanovsky, A. Maksimchuk, S. Kalmykov, G. Shvets, M. C. Downer, *Nature Phys.*, **2**, 749–753 (2006).
3. P. Dong, S. A. Reed, S. A. Yi, S. Kalmykov, Z. Y. Li, G. Shvets, N. H. Matlis, C. McGuffey, S. S. Bulanov, V. Chvykov, *New J. Phys.*, **12**, 045016 (2010).
4. Zhengyan Li, R. Zgadzaj, X. Wang, Y. Y. Chang, M. C. Downer, *Nature Commun.*, **5**, 3085 (2014).
5. Kenichi Yatsugi, Naoki Matsumoto, Takeshi Nagashima, Masanori Hangyo, *Appl. Phys. Lett.*, **98**, 212108 (2011).
6. M. Takeda, H. Ina, S. Kobayashi, *J. Opt. Soc. Am.*, **72**, 156 (1982).
7. C. Iaconis, I. A. Walmsley, *IEEE J. Quantum Electron.*, **35**, N 4, 501–509(1999).